Nine Men's Morris Cube Algorithms

Introduction

[Nine Men's Morris](#) is an abstract strategy board game for two players that emerged from the Roman Empire. The game is also known as *Nine Man Morris*, *Mill*, *Mills*, *Merels*, *Merelles*, and *Merrills* in English. Each player has nine pieces, or "men", which move among the board's twenty-four intersections. The object of the game is to leave the opposing player with fewer than three pieces or, as in checkers, no legal moves.

Placing Pieces

The game begins with an empty board. Players take turns placing their pieces on empty intersections. If a player is able to form a row of three pieces along one of the board's lines, he has a "mill" and may remove one of his opponent's pieces from the board; removed pieces may not be placed again. Players must remove any other pieces first before removing a piece from a formed mill. Once all eighteen pieces have been placed, players take turns moving.

Moving Pieces

To move, a player slides one of his pieces along a board line to an empty adjacent intersection. If he cannot do so, he has lost the game. As in the placement stage, a player who aligns three of his pieces on a board line has a mill and may remove one of his opponent's pieces, avoiding the removal of pieces in mills if at all possible. Any player reduced to two pieces is unable to remove any more opposing pieces and thus loses the game.

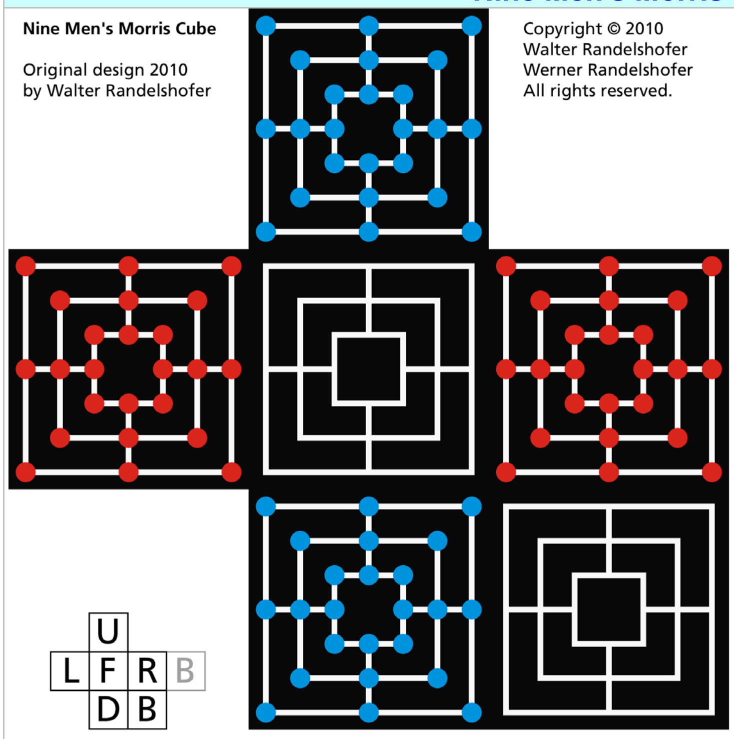
Strategy

At the beginning of the game, it is more important to place pieces in versatile locations rather than to try to form mills immediately and make the mistake of concentrating one's pieces in one area of the board. An ideal position, which typically results in a win, is to be able to shuttle one piece back and forth between two mills, removing a piece every turn.

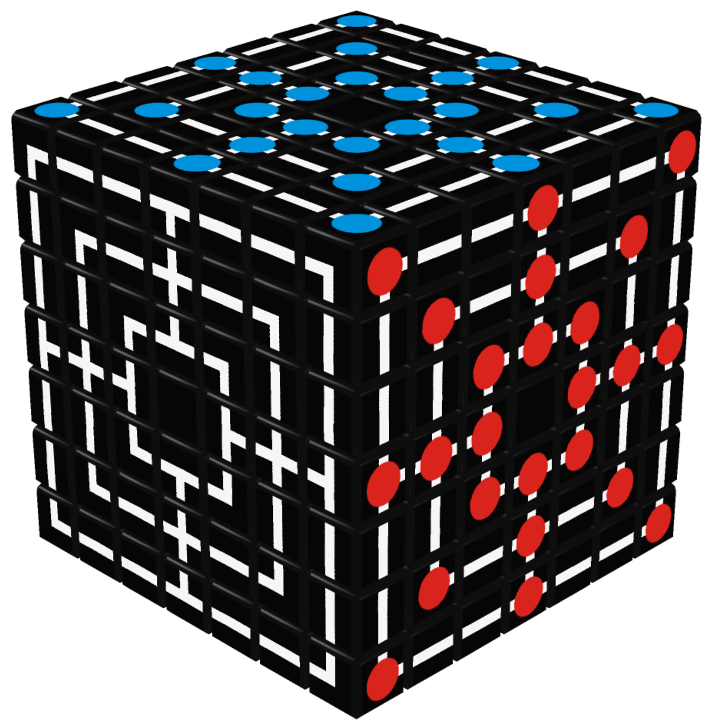
Nine Men's Morris Cube

A Nine men's Morris board game can be built from a texture laid on a 7x7x7 cube. By moving blank, blue or red stickers to the front face, two players can play the game. It is even possible to play against a computer.

Nine Men's Morris Cube – 7x7x7 Cube



Texture


7x7x7 Cube

Nine Men's Morris Cube Algorithms

Overview

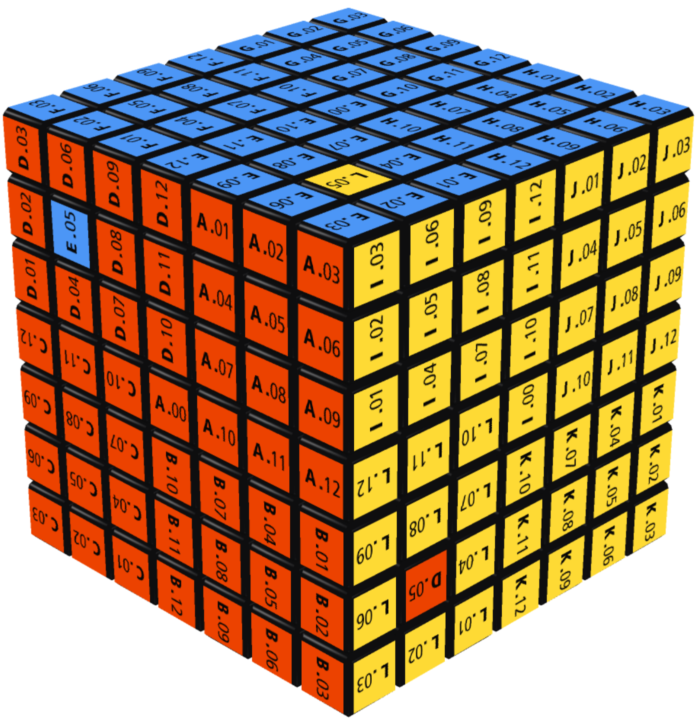
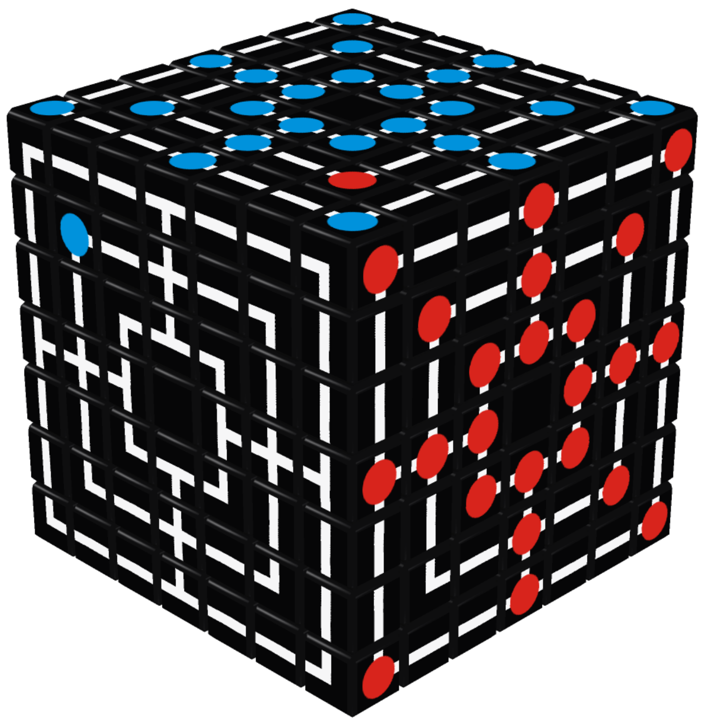
A set of algorithms is needed for moving cube pieces to the front face. By looking closely to the texture, it can be seen that algorithms are required only for corners, midges, corner-centers and midge-centers. Pieces are located on 3 concentric squares. Algorithms of the innermost square can then be obtained from those of the middle square simply by scaling slice moves. To lower the number of algorithms, only corner twists and 3-cycles of midges and centers have been considered and all algorithms computed using the [Algorithm Picker](#) tool. By using incremental permutations, it is no longer necessary to save the state of the whole cube between each player's turn. There are only 3 states to be considered per sticker because a front face sticker can be either blank or blue or red. Depending on the previous state, the present state can be reached either by an Identity perm, a Perm+ or a Perm–. Moreover, if Alg is the algorithm of Perm+, then Alg' is the algorithm of Perm–. Thus 1 algorithm only is needed per case, as shown in the tables below.

Incremental Permutations

Previous State	Present State	Incremental Permutation	Algorithm
0	0	Identity	No Move
	1	Perm+	Alg
	2	Perm–	Alg'
1	0	Perm–	Alg'
	1	Identity	No Move
	2	Perm+	Alg
2	0	Perm+	Alg
	1	Perm–	Alg'
	2	Identity	No Move

Example: 3-Cycle of Corner-Centers

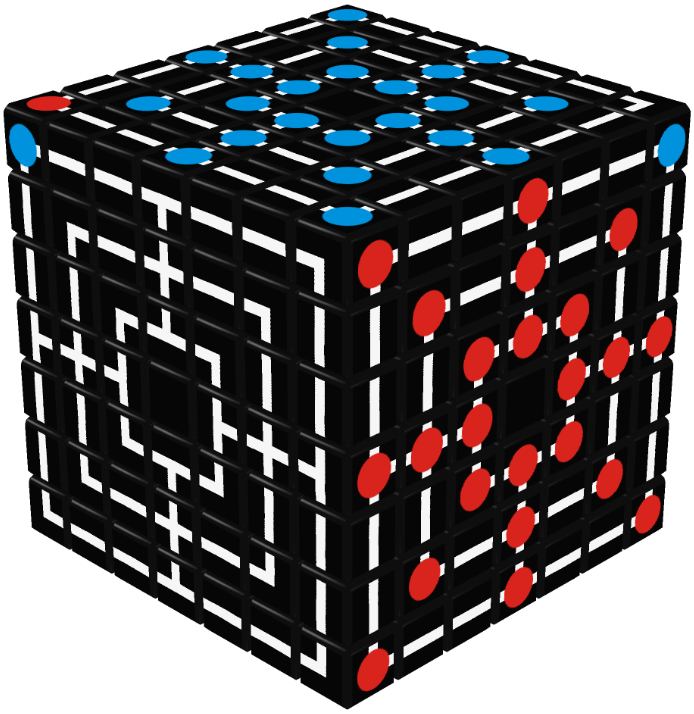
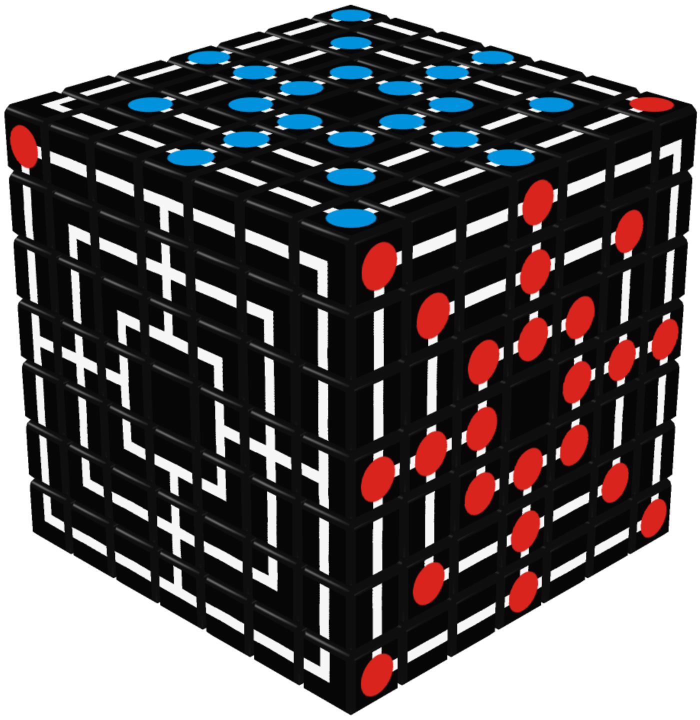
Previous State	Present State	Incremental Permutation	Algorithm – 8 moves
0	0	Identity	No Move
	1	(D L E)	ND' F' ND NF ND' F ND NF'
	2	(D E L)	(ND' F' ND NF ND' F ND NF')'
1	0	(D E L)	(ND' F' ND NF ND' F ND NF')'
	1	Identity	No Move
	2	(D L E)	ND' F' ND NF ND' F ND NF'
2	0	(D L E)	ND' F' ND NF ND' F ND NF'
	1	(D E L)	(ND' F' ND NF ND' F ND NF')'
	2	Identity	No Move

Orbit Cube – (D L E) Permutation	Nine Men's Morris Cube – (D L E) Permutation
	

Corner Algorithms

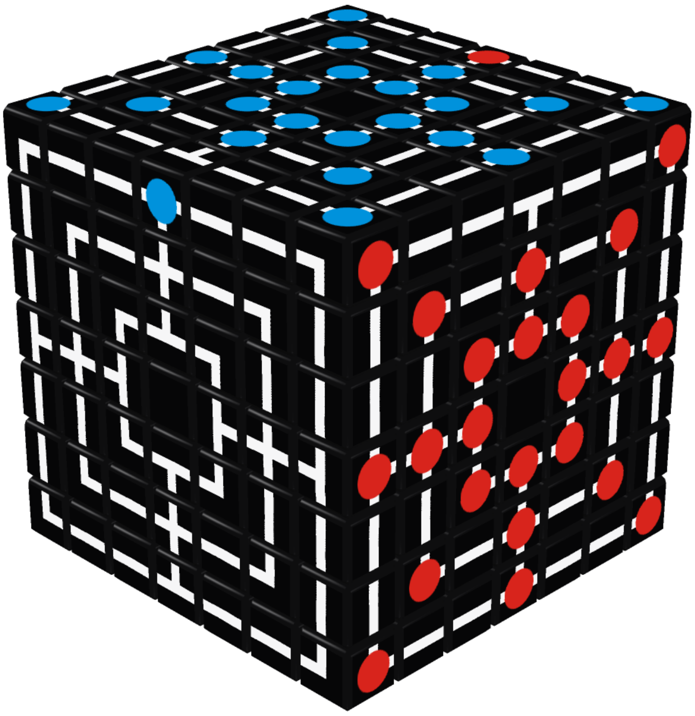
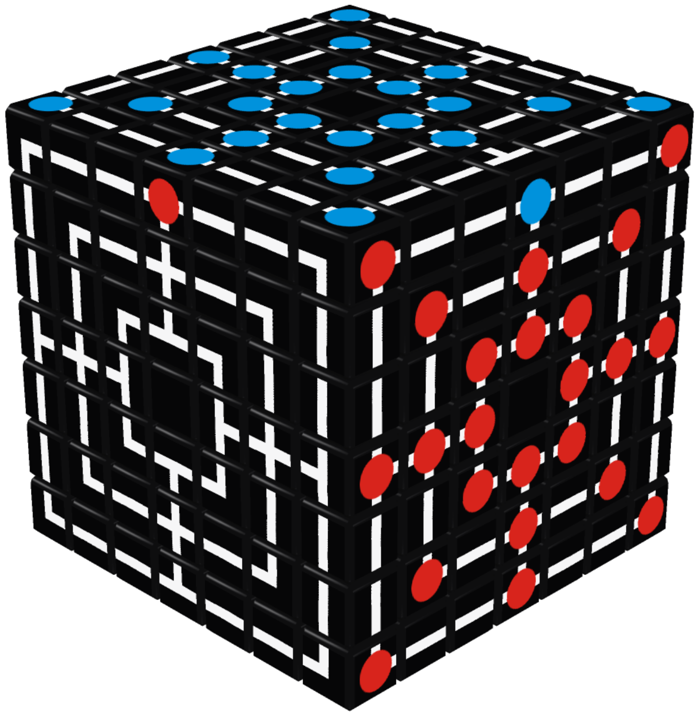
Each of the 4 corners of face F can be paired up with 1 of the 4 corners of face B. By twisting each corner of a pair in opposite directions, front face corner stickers can then be set to either blank, blue or red. Pairs of corners can be selected so as to give the shortest algorithms, which are 12 moves long.

Corner Twist Algorithms		
Corner – Front Face	Paired Corner – Back Face	Algorithm – 12 moves
(D, O, F)	(W, J, H)	R F' D2 F R' U2 R F' D2 F R' U2
(A, E, I)	(V, T, K)	U B' L2 B U' R2 U B' L2 B U' R2
(B, L, S)	(U, M, Q)	D2 B' R U2 R' B D2 B' R U2 R' B
(C, R, P)	(X, G, N)	L2 F' U R2 U' F L2 F' U R2 U' F

Example: Corner (D, O, F)	
Blue	Red
	
R F' D2 F R' U2 R F' D2 F R' U2	(R F' D2 F R' U2 R F' D2 F R' U2)'

Midge 3-Cycle Algorithms

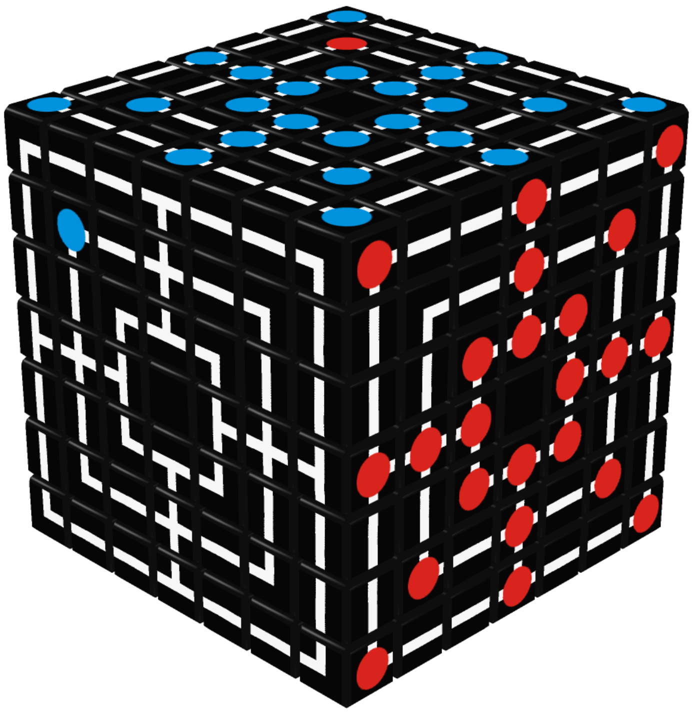
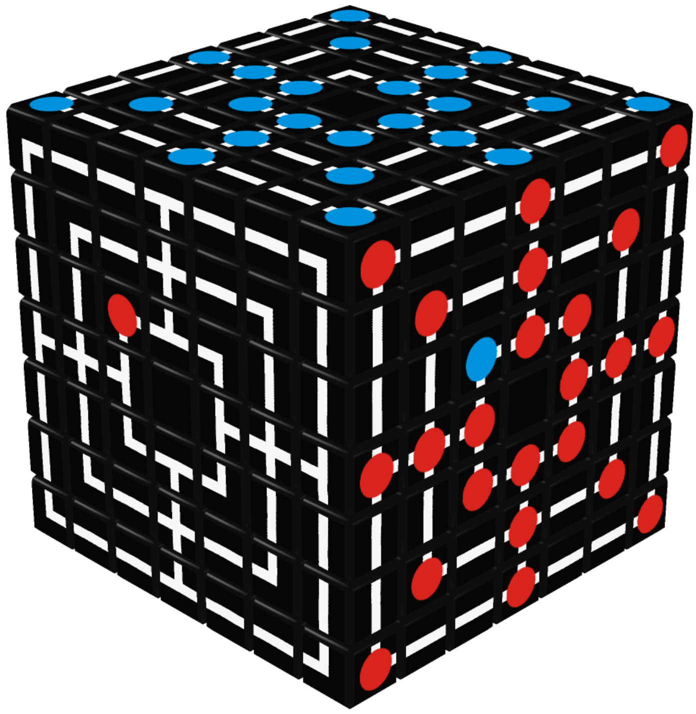
Each of the 4 midges of face F can be paired up with 2 other midges to built a 3-cycle. By cycling the 3 midges, front face midge stickers can then be set to either blank, blue or red. The 3-cycles of midges can be selected so as to give the shortest algorithms, which are 8 moves long.

Midge 3-Cycle Algorithms	
Midge Sticker 3-Cycle	Algorithm – 8 moves
(D I G)	B' R B MR B' R' B MR'
(A J S)	MU' B' D B MU B' D' B
(B P T)	B' L B MR' B' L' B MR
(C M F)	MU B' U B MU' B' U' B
Example: Midge (D, E)	
Blue	Red
	
B' R B MR B' R' B MR'	(B' R B MR B' R' B MR')'

Corner-Center 3-Cycle Algorithms

Each of the 4 corner-centers of face F can be paired up with 2 other corner-centers to built a 3-cycle. By cycling the 3 centers, front face center stickers can then be set to either blank, blue or red. The 3-cycles of centers can be selected so as to give the shortest algorithms, which are 8 moves long.

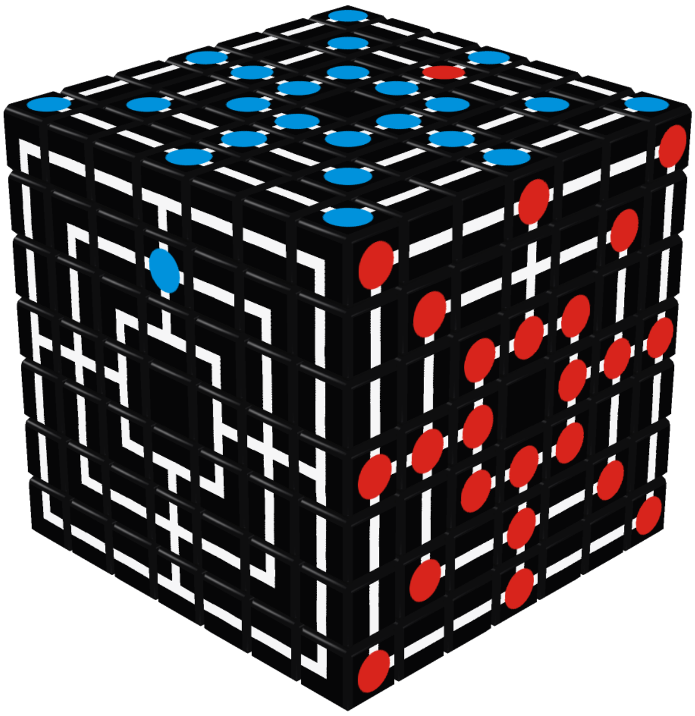
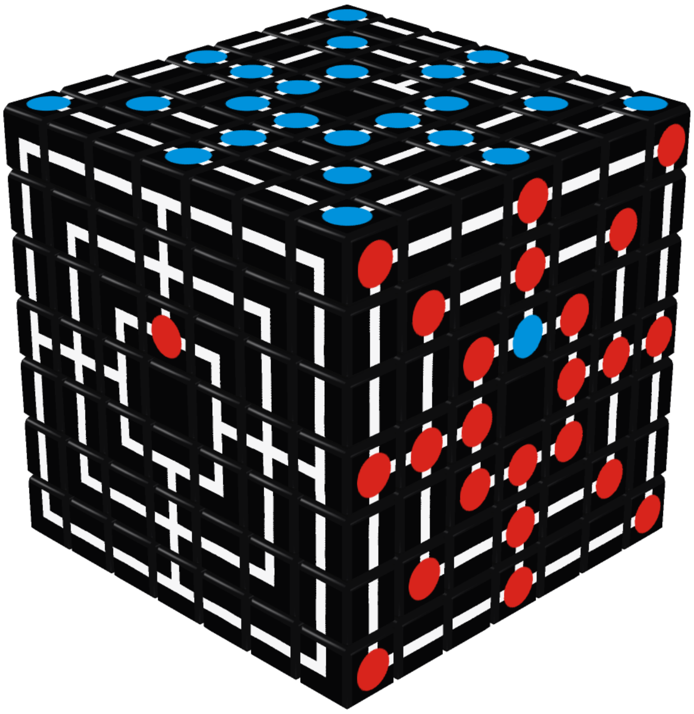
Notice that 3-cycles of corner-centers and midges are the same, eg. (D I G) and that Orbit 07 algorithms are scaled versions of Orbit 05 algorithms.

Corner-Center 3-Cycle Algorithms	
Corner-Center Sticker 3-Cycle – Orbit 05	Algorithm – 8 moves
(D I G)	NL B NU B' NL' B NU' B'
(A J S)	NR B' NU B NR' B' NU' B
(B P T)	NR B ND B' NR' B ND' B'
(C M F)	NL B' ND B NL' B' ND' B
Corner-Center Sticker 3-Cycle – Orbit 07	Algorithm – 8 moves
(D I G)	N3L B N3U B' N3L' B N3U' B'
(A J S)	N3R B' N3U B N3R' B' N3U' B
(B P T)	N3R B N3D B' N3R' B N3D' B'
(C M F)	N3L B' N3D B N3L' B' N3D' B
Example: Corner-Center D	
Blue – Orbit 05	Red – Orbit 07
	
NL B NU B' NL' B NU' B'	(N3L B N3U B' N3L' B N3U' B')'

Midge-Center 3-Cycle Algorithms

Each of the 4 midge-centers of face F can be paired up with 2 other midge-centers to built a 3-cycle. By cycling the 3 centers, front face center stickers can then be set to either blank, blue or red. The 3-cycles of centers can be selected so as to give the shortest algorithms, which are 8 moves long.

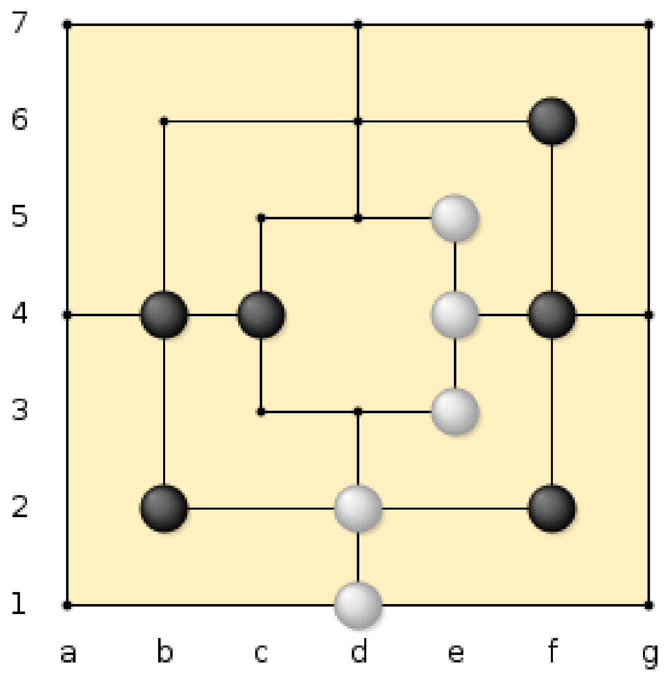
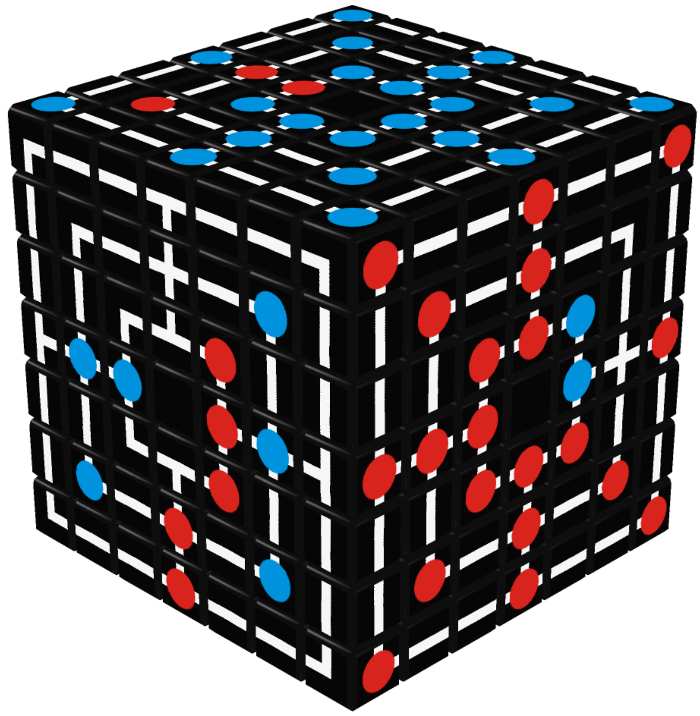
Notice that 3-cycles of midge-centers, corner-centers and midges are the same, eg. (D I G) and that Orbit 10 algorithms are scaled versions of Orbit 11 algorithms.

Midge-Center 3-Cycle Algorithms	
Midge-Center Sticker 3-Cycle – Orbit 11	Algorithm – 8 moves
(D I G)	B' MR' B NU B' MR B NU'
(A J S)	B' NR B MU B' NR' B MU'
(B P T)	B' MR B ND B' MR' B ND'
(C M F)	B' NL B MU' B' NL' B MU
Midge-Center Sticker 3-Cycle – Orbit 10	Algorithm – 8 moves
(D I G)	B' MR' B N3U B' MR B N3U'
(A J S)	B' N3R B MU B' N3R' B MU'
(B P T)	B' MR B N3D B' MR' B N3D'
(C M F)	B' N3L B MU' B' N3L' B MU
Example: Midge-Center D	
Blue – Orbit 11	Red – Orbit 10
	
B' MR' B NU B' MR B NU'	(B' MR' B N3U B' MR B N3U')'

Algorithms – Example

Pieces can be moved to the front face one at a time and independently from other pieces. Blank, blue or red stickers are selected using 3-cycles or corner twists. Algorithms can be applied in any order because they don't move the same pieces.

In a normal play, a blue sticker is moved first by the blue player, then a red sticker is moved by the red player, so that blue and red stickers are moved alternately until one of the two players wins. An example of a play is shown in the table below where 11 algorithms have been used for a total of 88 moves.

Algorithms – Example	
Blue Stickers – Face F	**Algorithms**
Corner-center f6	NR B' NU B NR' B' NU' B
Corner-center f2	NR B ND B' NR' B ND' B'
Corner-center b2	NL B' ND B NL' B' ND' B
Midge-center f4	B' NR B MU B' NR' B MU'
Midge-center b4	B' NL B MU' B' NL' B MU
Midge-center c4	B' N3L B MU' B' N3L' B MU
Red Stickers – Face F	**Algorithms**
Midge d1	(B' L B MR' B' L' B MR)'
Midge-center d2	(B' MR B ND B' MR' B ND')'
Corner-center e5	(N3R B' N3U B N3R' B' N3U' B)'
Corner-center e3	(N3R B N3D B' N3R' B N3D' B')'
Midge-center e4	(B' N3R B MU B' N3R' B MU')'
Nine Men's Morris – Example	
Board Game	**Game Cube**
	
Even if it's black's turn, white can win by moving from e3 to d3 and back again twice, removing a black piece each time a row of three is formed.	Pieces can be moved to the front face one at a time and independently from other pieces. Blank, blue or red stickers are selected using 3-cycles.

Sticker Numbering

Front face stickers are numbered from 0 to 23, as shown on the 7x7x7 texture below. Face F state is then defined as an array of 24 items. Each item can take 3 values, depending on sticker color:

0 (Blank), 1 (Blue) and 2 (Red)

The state array of the preceeding example can then be written as:

[0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 1, 1, 1, 2, 1, 1, 0, 0, 2, 2, 2, 0, 0, 1]
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23