True Centers

Introduction

True centers only exist for odd-order cubes, eg. 3x3x3, 5x5x5 and 7x7x7 cubes. Compared to other types of cube pieces, ie. corners, midges, edges or centers, true centers are more restricted in their moves. It should be noted that only 4-spot or 6-spot patterns do exist in the case of true center *legal* moves.

Permutation of True Centers

Due to mechanical restrictions, true centers do not behave exactly as other centers do. But the same permutation laws apply to them with the added restriction of orientation parity. Main restrictions are listed below:

1- Only 4-spot and 6-spot true center permutations are legal
2- Adjacent centers will always stay adjacent through any legal move
3- Opposed centers will always stay opposed through any legal move
4- For a *solved* regular cube, the sum of the orientations of all centers will always be equal either to 0° or 180° modulo 360°

Cycle structures of true center permutations, shown in the table below, are either of even or of odd parity. Odd parity permutations imply that some other pieces are also messed up, eg. midges.

True Centers – Cycle Structures			
N-spot	Cycle Structures	Notes	Permutation Parity
4-spot	2 2-cycles	2-cycles: 2 *opposed* centers	even
4-spot	4-cycle	–	odd
6-spot	2 3-cycles	3-cycles: 3 *adjacent* centers	even
6-spot	3 2-cycles	–	odd

Number of Permutation/Orientation Cases

For 4-spot patterns, there are 3 permutation cases of even parity and 6 of odd parity, for a total of 9 cases. For each of these cases, there are also 4096 distinct orientations, where each center orientation can be equal to 0°, 90°, 180° or 270°, from which 2048 are of even parity and 2048 of odd parity. Orientations are of even parity if the sum of all 6 center angular rotations is equal either to 0° or 180° modulo 360°, and of odd parity otherwise.

The total number of cases for 4-spot patterns is then given by:

9x4096 = 36864

For 6-spot patterns, there are 8 permutation cases of even parity and 6 of odd parity, for a total of 14 cases. For each of these cases, there are also 4096 distinct orientations.

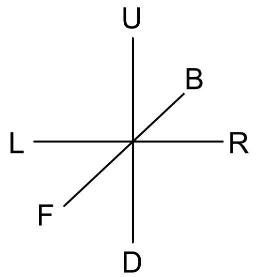
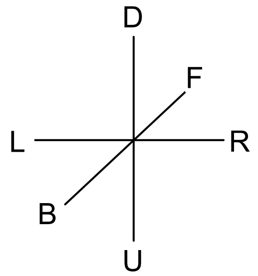
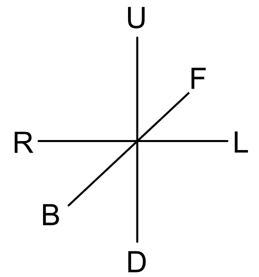
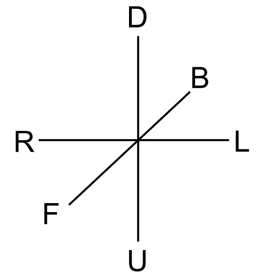
The total number of cases for 6-spot patterns is then given by:

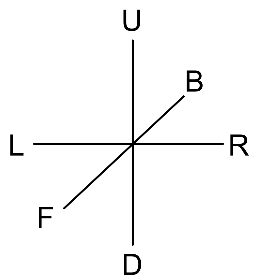
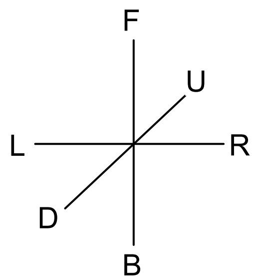
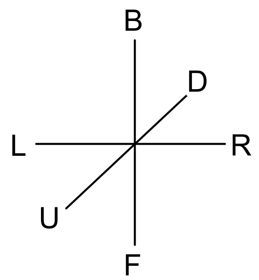
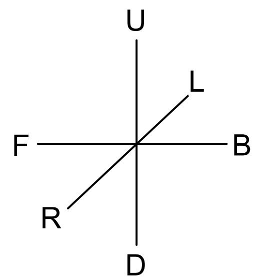
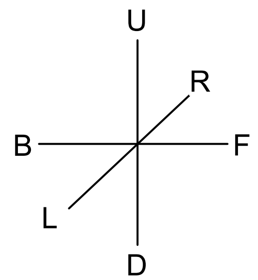
14x4096 = 57344

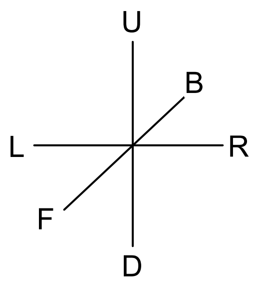
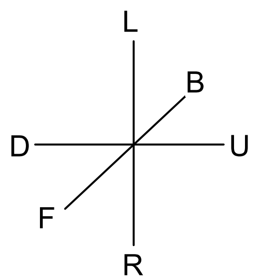
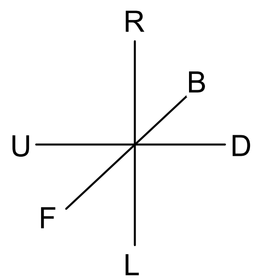
True Center Permutations

4-Spot Pattern

There are 9 cases of 4-spot patterns.

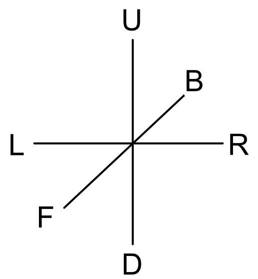
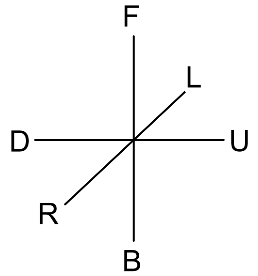
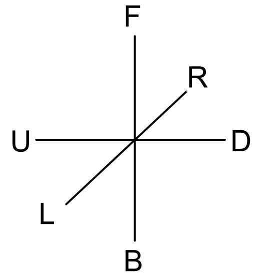
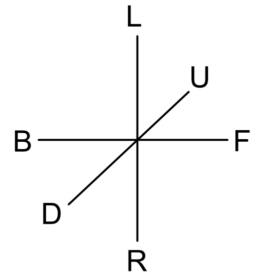
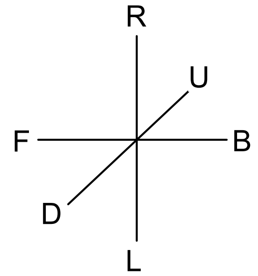
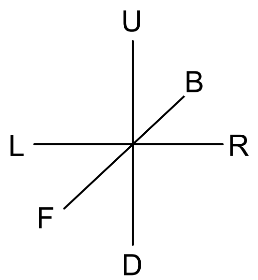
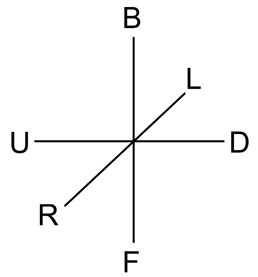
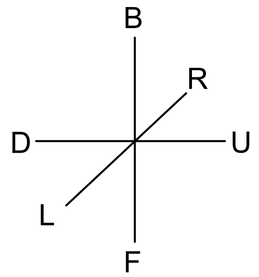
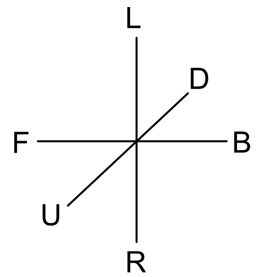
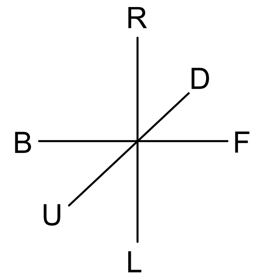
True Center Permutations – 4-Spot Patterns – 2 2-Cycles				
Reference	CR2	CU2	CF2	
				
Permutations - Let	(F B) (U D)	(F B) (R L)	(R L) (U D)	
Permutations - Num	(0 5) (2 4)	(0 5) (1 3)	(1 3) (2 4)	
Cycles	2 2-cycles	2 2-cycles	2 2-cycles	
Permutation Parity	even	even	even	

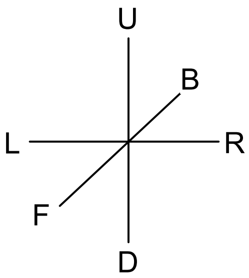
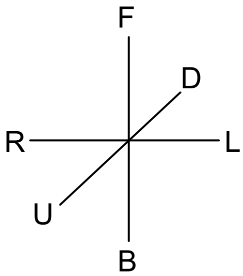
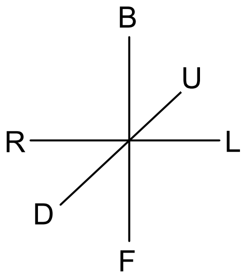
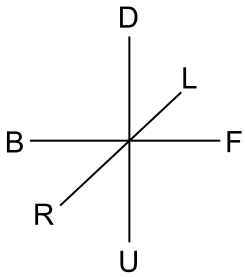
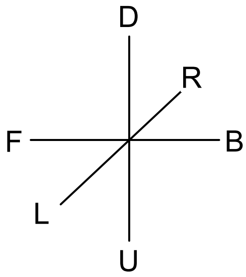
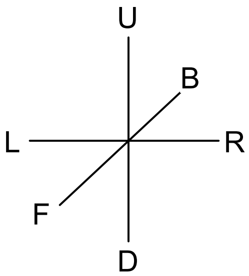
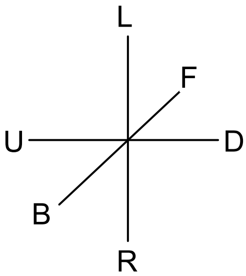
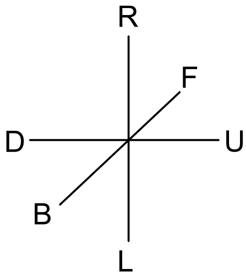
True Center Permutations – 4-Spot Pattern – 4-Cycles				
Reference	CR	CR'	CU	CU'
				
Permutations - Let	(F U B D)	(F D B U)	(F L B R)	(F R B L)
Permutations - Num	(0 2 5 4)	(0 4 5 2)	(0 3 5 1)	(0 1 5 3)
Cycles	4-cycle	4-cycle	4-cycle	4-cycle
Permutation Parity	odd	odd	odd	odd

True Center Permutations – 4-Spot Pattern – 4-Cycles				
Reference	CF	CF'		
				
Permutations - Let	(R D L U)	(R U L D)		
Permutations - Num	(1 4 3 2)	(1 2 3 4)		
Cycles	4-cycle	4-cycle		
Permutation Parity	odd	odd		

6-Spot Pattern

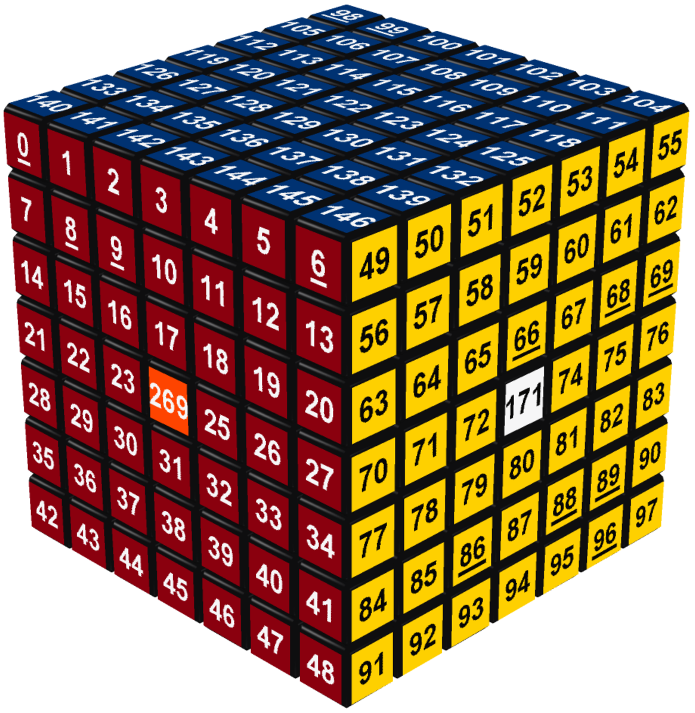
There are 14 cases of 6-spot patterns.

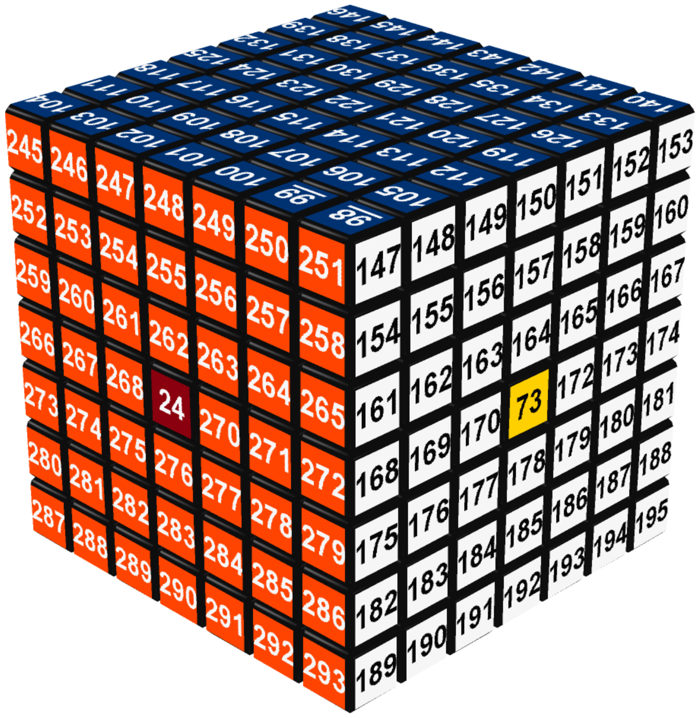
True Center Permutations – 6-Spot Pattern – 2 3-Cycles				
Reference	CR CU	CR CU'	CR CF	CR CF'
				
Permutations - Let	(F U R) (L B D)	(F U L) (R B D)	(F R D) (U B L)	(F L D) (R U B)
Permutations - Num	(0 2 1) (3 5 4)	(0 2 3) (1 5 4)	(0 1 4) (2 5 3)	(0 3 4) (1 2 5)
Cycles	2 3-cycles	2 3-cycles	2 3-cycles	2 3-cycles
Permutation Parity	even	even	even	even
True Center Permutations – 6-Spot Pattern – 2 3-Cycles				
Reference	CR' CU	CR' CU'	CR' CF	CR' CF'
				
Permutations - Let	(F D R) (U L B)	(F D L) (R B U)	(F L U) (R D B)	(F R U) (L D B)
Permutations - Num	(0 4 1) (2 3 5)	(0 4 3) (1 5 2)	(0 3 2) (1 4 5)	(0 1 2) (3 4 5)
Cycles	2 3-cycles	2 3-cycles	2 3-cycles	2 3-cycles
Permutation Parity	even	even	even	even

True Center Permutations – 6-Spot Pattern – 3 2-Cycles				
Reference	CR CU2	CR CF2	CR2 CU	CR2 CU'
				
Permutations - Let	(F U) (R L) (D B)	(F D) (U B) (R L)	(F R) (U D) (L B)	(F L) (R B) (U D)
Permutations - Num	(0 2) (1 3) (4 5)	(0 4) (2 5) (1 3)	(0 1) (2 4) (3 5)	(0 3) (1 5) (2 4)
Cycles	3 2-cycles	3 2-cycles	3 2-cycles	3 2-cycles
Permutation Parity	odd	odd	odd	odd
True Center Permutations – 6-Spot Pattern – 3 2-Cycles				
Reference	CR2 CF	CR2 CF'		
				
Permutations - Let	(F B) (R D) (U L)	(F B) (R U) (L D)		
Permutations - Num	(0 5) (1 4) (2 3)	(0 5) (1 2) (3 4)		
Cycles	3 2-cycles	3 2-cycles		
Permutation Parity	odd	odd		

Algorithms

4-Spot Pattern – 2 2-Cycles

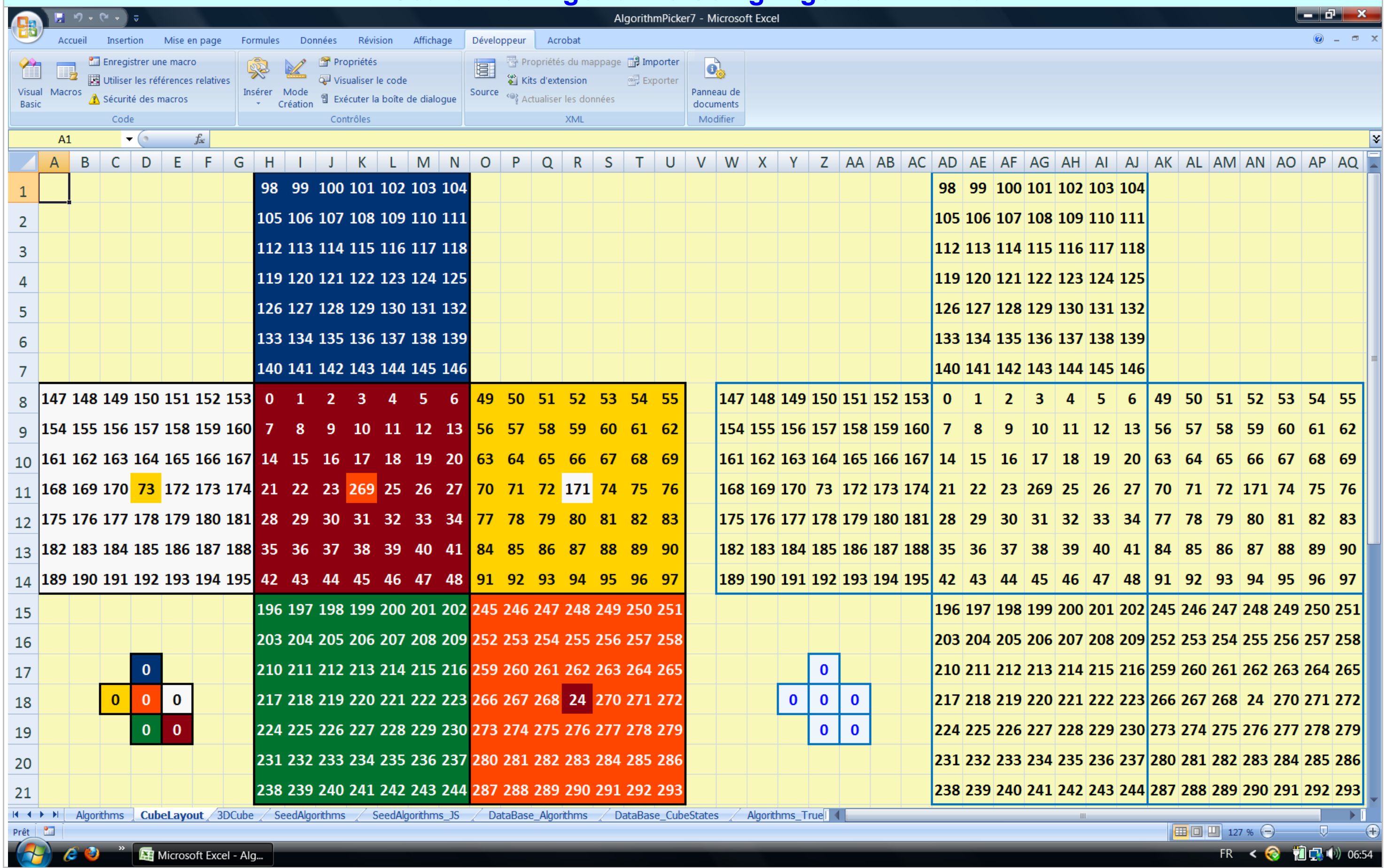




Permutation: (F B) (R L) – Orientations: 0°

MR' MU MR' MU2 MR MU MR

Search for Algorithms Using Algorithm Finder



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1								98	99	100	101	102	103	104							
2								105	106	107	108	109	110	111							
3								112	113	114	115	116	117	118							
4								119	120	121	122	123	124	125							
5								126	127	128	129	130	131	132							
6								133	134	135	136	137	138	139							
7								140	141	142	143	144	145	146							
8	147	148	149	150	151	152	153	0	1	2	3	4	5	6	49	50	51	52	53	54	55
9	154	155	156	157	158	159	160	7	8	9	10	11	12	13	56	57	58	59	60	61	62
10	161	162	163	164	165	166	167	14	15	16	17	18	19	20	63	64	65	66	67	68	69
11	168	169	170	73	172	173	174	21	22	23	269	25	26	27	70	71	72	171	74	75	76
12	175	176	177	178	179	180	181	28	29	30	31	32	33	34	77	78	79	80	81	82	83
13	182	183	184	185	186	187	188	35	36	37	38	39	40	41	84	85	86	87	88	89	90
14	189	190	191	192	193	194	195	42	43	44	45	46	47	48	91	92	93	94	95	96	97
15								196	197	198	199	200	201	202	245	246	247	248	249	250	251
16								203	204	205	206	207	208	209	252	253	254	255	256	257	258
17				0				210	211	212	213	214	215	216	259	260	261	262	263	264	265
18			0	0	0			217	218	219	220	221	222	223	266	267	268	24	270	271	272
19				0	0			224	225	226	227	228	229	230	273	274	275	276	277	278	279
20								231	232	233	234	235	236	237	280	281	282	283	284	285	286
21								238	239	240	241	242	243	244	287	288	289	290	291	292	293

	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ
1									98	99	100	101	102	103	104							
2									105	106	107	108	109	110	111							
3									112	113	114	115	116	117	118							
4									119	120	121	122	123	124	125							
5									126	127	128	129	130	131	132							
6									133	134	135	136	137	138	139							
7									140	141	142	143	144	145	146							
8		147	148	149	150	151	152	153	0	1	2	3	4	5	6	49	50	51	52	53	54	55
9		154	155	156	157	158	159	160	7	8	9	10	11	12	13	56	57	58	59	60	61	62
10		161	162	163	164	165	166	167	14	15	16	17	18	19	20	63	64	65	66	67	68	69
11		168	169	170	73	172	173	174	21	22	23	269	25	26	27	70	71	72	171	74	75	76
12		175	176	177	178	179	180	181	28	29	30	31	32	33	34	77	78	79	80	81	82	83
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14		189	190	191	192	193	194	195	42	43	44	45	46	47	48	91	92	93	94	95	96	97
15									196	197	198	199	200	201	202	245	246	247	248	249	250	251
16									203	204	205	206	207	208	209	252	253	254	255	256	257	258
17					0				210	211	212	213	214	215	216	259	260	261	262	263	264	265
18				0	0	0			217	218	219	220	221	222	223	266	267	268	24	270	271	272
19					0	0			224	225	226	227	228	229	230	273	274	275	276	277	278	279
20									231	232	233	234	235	236	237	280	281	282	283	284	285	286
21									238	239	240	241	242	243	244	287	288	289	290	291	292	293

Template: [MX, MY] [MZ, MP]

All true centers orientations are set to 0°