Cube Symmetry

Basic Theory

A cube has rotation axes of 3 kinds:

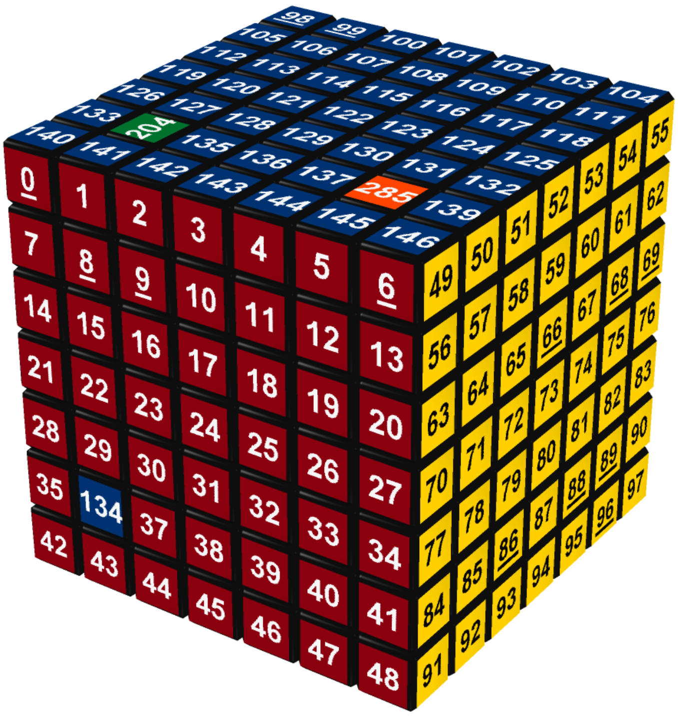
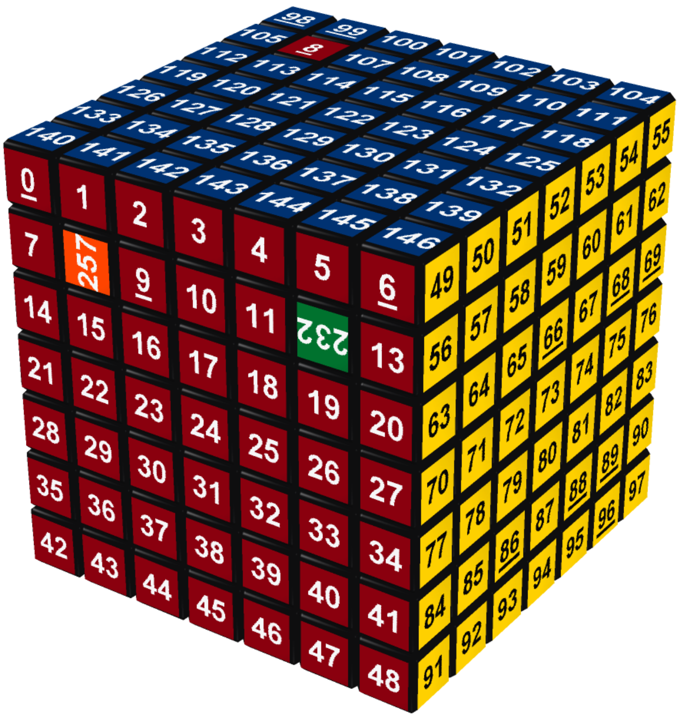
- 3 axes through opposed face centers, with 3 rotation angles of 90°, 180° and 270° (or -90°)
- 4 axes through opposed corners, with 2 rotation angles of +120° and +240° (or -120°)
- 6 axes through opposed edges, with 1 angle of rotation of 180°

So that the cube presents 3x3 + 4x2 + 6x1 = 23 proper rotations. Of course there is also the identity rotation, so that the total number of all rotations is actually equal to 24. By combining each of these 24 rotational (or orientation-preserving) symmetries with a point reflection, 24 additional transformations are obtained. The complete set of cube symmetries thus includes 48 transformations. Each of these can be defined by a specific transformation of the initial set of 6 cube faces (F, R, U, L, D, B) into another set of primed/unprimed faces, such as a cube rotation through opposed edges FD – UB, followed by a point reflection:

(F, R, U, L, D, B) → (U', R', F', L', B', D')

Symmetry and Algorithms

This basic theory of cube symmetry can be applied to finding new algorithms, given an already known algorithm. If an algorithm is known to move some pieces on a cube, then by applying one of these transformations, another algorithm is obtained that will move other pieces, while preserving the overall cube symmetry. The example below shows how a 5-cycle of corner-centers can be transformed into another 5-cycle, just by applying the above-mentioned transformation.

Cube Rotation (FD – UB Axis) + Point Reflection		
Corner-Centers 5-cycle		
		
[NF L NF, NL']	F→ U', L→ L'	[NU' L' NU', NL]

By inverting each of the 48 transformed algorithms, an additional set of 48 algorithms is obtained, thus leading to a total of 48 + 48 = 96 algorithms. This number of algorithms can be further increased by adding setup moves to each of the already available 96 algorithms (setup moves are usually 1, 2 or 3 moves long):

(Setup) (Algorithm) (Setup)'

Tables of 48 Transformations

Cube Rotations: 24

Identity Rotation
NoMove
F→F
R→R
U→U
L→L
D→D
B→B
#0

Cube Rotations (3 Axes)								
F – B Axis			R – L Axis			U – D Axis		
+90°	+180°	+270°	+90°	+180°	+270°	+90°	+180°	+270°
CF	CF2	CF'	CR	CR2	CR'	CU	CU2	CU'
F→F	F→F	F→F	F→U	F→B	F→D	F→L	F→B	F→R
R→D	R→L	R→U	R→R	R→R	R→R	R→F	R→L	R→B
U→R	U→D	U→L	U→B	U→D	U→F	U→U	U→U	U→U
L→U	L→R	L→D	L→L	L→L	L→L	L→B	L→R	L→F
D→L	D→U	D→R	D→F	D→U	D→B	D→D	D→D	D→D
B→B	B→B	B→B	B→D	B→F	B→U	B→R	B→F	B→L
#1	#2	#3	#4	#5	#6	#7	#8	#9

Cube Rotations (4 Axes)							
FRD – ULB Axis		FLD – RUB Axis		FRU – LDB Axis		FUL – RDB Axis	
-120°	+120°	-120°	+120°	-120°	+120°	-120°	+120°
CF' CR'	CF CU'	CF' CU	CF CR'	CF' CU'	CF CR	CF' CR	CF CU
F→D	F→R	F→L	F→D	F→R	F→U	F→U	F→L
R→F	R→D	R→U	R→B	R→U	R→F	R→B	R→D
U→L	U→B	U→B	U→R	U→F	U→R	U→L	U→F
L→B	L→U	L→D	L→F	L→D	L→B	L→F	L→U
D→R	D→F	D→F	D→L	D→B	D→L	D→R	D→B
B→U	B→L	B→R	B→U	B→L	B→D	B→D	B→R
#10	#11	#12	#13	#14	#15	#16	#17

Cube Rotations (6 Axes)					
FD – UB Axis	FU – DB Axis	RD – UL Axis	RU – LD Axis	FR – LB Axis	FL – RB Axis
+180°	+180°	+180°	+180°	+180°	+180°
CF2 CR'	CR CU2	CF CU2	CF' CU2	CU' CR2	CF2 CU
F→D	F→U	F→B	F→B	F→R	F→L
R→L	R→L	R→D	R→U	R→F	R→B
U→B	U→F	U→L	U→R	U→D	U→D
L→R	L→R	L→U	L→D	L→B	L→F
D→F	D→B	D→R	D→L	D→U	D→U
B→U	B→D	B→F	B→F	B→L	B→R
#18	#19	#20	#21	#22	#23

Cube Rotations + Point Reflection: 24

Point Reflection
PR
F→B'
R→L'
U→D'
L→R'
D→U'
B→F'
#24

Cube Rotations (3 Axes) + Point Reflection

F – B Axis			R – L Axis			U – D Axis		
+90°	+180°	+270°	+90°	+180°	+270°	+90°	+180°	+270°
CF PR	CF2 PR	CF' PR	CR PR	CR2 PR	CR' PR	CU PR	CU2 PR	CU' PR
F→F→B'	F→F→B'	F→F→B'	F→U→D'	F→B→F'	F→D→U'	F→L→R'	F→B→F'	F→R→L'
R→D→U'	R→L→R'	R→U→D'	R→R→L'	R→R→L'	R→R→L'	R→F→B'	R→L→R'	R→B→F'
U→R→L'	U→D→U'	U→L→R'	U→B→F'	U→D→U'	U→F→B'	U→U→D'	U→U→D'	U→U→D'
L→U→D'	L→R→L'	L→D→U'	L→L→R'	L→L→R'	L→L→R'	L→B→F'	L→R→L'	L→F→B'
D→L→R'	D→U→D'	D→R→L'	D→F→B'	D→U→D'	D→B→F'	D→D→U'	D→D→U'	D→D→U'
B→B→F'	B→B→F'	B→B→F'	B→D→U'	B→F→B'	B→U→D'	B→R→L'	B→F→B'	B→L→R'
#25	#26	#27	#28	#29	#30	#31	#32	#33

Cube Rotations (4 Axes) + Point Reflection

FRD – ULB Axis		FLD – RUB Axis		FRU – LDB Axis		FUL – RDB Axis	
-120°	+120°	-120°	+120°	-120°	+120°	-120°	+120°
CF' CR' PR	CF CU' PR	CF' CU PR	CF CR' PR	CF' CU' PR	CF CR PR	CF' CR PR	CF CU PR
F→D→U'	F→R→L'	F→L→R'	F→D→U'	F→R→L'	F→U→D'	F→U→D'	F→L→R'
R→F→B'	R→D→U'	R→U→D'	R→B→F'	R→U→D'	R→F→B'	R→B→F'	R→D→U'
U→L→R'	U→B→F'	U→B→F'	U→R→L'	U→F→B'	U→R→L'	U→L→R'	U→F→B'
L→B→F'	L→U→D'	L→D→U'	L→F→B'	L→D→U'	L→B→F'	L→F→B'	L→U→D'
D→R→L'	D→F→B'	D→F→B'	D→L→R'	D→B→F'	D→L→R'	D→R→L'	D→B→F'
B→U→D'	B→L→R'	B→R→L'	B→U→D'	B→L→R'	B→D→U'	B→D→U'	B→R→L'
#34	#35	#36	#37	#38	#39	#40	#41

Cube Rotations (6 Axes) + Point Reflection

FD – UB Axis	FU – DB Axis	RD – UL Axis	RU – LD Axis	FR – LB Axis	FL – RB Axis
+180°	+180°	+180°	+180°	+180°	+180°
CF2 CR' PR	CR CU2 PR	CF CU2 PR	CF' CU2 PR	CU' CR2 PR	CF2 CU PR
F→D→U'	F→U→D'	F→B→F'	F→B→F'	F→R→L'	F→L→R'
R→L→R'	R→L→R'	R→D→U'	R→U→D'	R→F→B'	R→B→F'
U→B→F'	U→F→B'	U→L→R'	U→R→L'	U→D→U'	U→D→U'
L→R→L'	L→R→L'	L→U→D'	L→D→U'	L→B→F'	L→F→B'
D→F→B'	D→B→F'	D→R→L'	D→L→R'	D→U→D'	D→U→D'
B→U→D'	B→D→U'	B→F→B'	B→F→B'	B→L→R'	B→R→L'
#42	#43	#44	#45	#46	#47

Tables of 48 Inverse Transformations

Cube Rotations Inverted: 24

Identity Rotation
NoMove
F→F
R→R
U→U
L→L
D→D
B→B
#0'

Cube Rotations Inverted (3 Axes)								
F – B Axis			R – L Axis			U – D Axis		
CF'	CF2	CF	CR'	CR2	CR	CU'	CU2	CU
F→F	F→F	F→F	F→D	F→B	F→U	F→R	F→B	F→L
R→U	R→L	R→D	R→R	R→R	R→R	R→B	R→L	R→F
U→L	U→D	U→R	U→F	U→D	U→B	U→U	U→U	U→U
L→D	L→R	L→U	L→L	L→L	L→L	L→F	L→R	L→B
D→R	D→U	D→L	D→B	D→U	D→F	D→D	D→D	D→D
B→B	B→B	B→B	B→U	B→F	B→D	B→L	B→F	B→R
#1'	#2'	#3'	#4'	#5'	#6'	#7'	#8'	#9'

Cube Rotations Inverted (4 Axes)							
FRD – ULB Axis		FLD – RUB Axis		FRU – LDB Axis		FUL – RDB Axis	
CR CF	CU CF'	CU' CF	CR CF'	CU CF	CR' CF'	CR' CF	CU' CF'
F→R	F→D	F→D	F→L	F→U	F→R	F→L	F→U
R→D	R→F	R→B	R→U	R→F	R→U	R→D	R→B
U→B	U→L	U→R	U→B	U→R	U→F	U→F	U→L
L→U	L→B	L→F	L→D	L→B	L→D	L→U	L→F
D→F	D→R	D→L	D→F	D→L	D→B	D→B	D→R
B→L	B→U	B→U	B→R	B→D	B→L	B→R	B→D
#10'	#11'	#12'	#13'	#14'	#15'	#16'	#17'

Cube Rotations Inverted (6 Axes)					
FD – UB Axis	FU – DB Axis	RD – UL Axis	RU – LD Axis	FR – LB Axis	FL – RB Axis
CR CF2	CU2 CR'	CU2 CF'	CU2 CF	CR2 CU	CU' CF2
F→D	F→U	F→B	F→B	F→R	F→L
R→L	R→L	R→D	R→U	R→F	R→B
U→B	U→F	U→L	U→R	U→D	U→D
L→R	L→R	L→U	L→D	L→B	L→F
D→F	D→B	D→R	D→L	D→U	D→U
B→U	B→D	B→F	B→F	B→L	B→R
#18'	#19'	#20'	#21'	#22'	#23'

Point Reflection Inverted + Cube Rotations Inverted: 24

Point Reflection Inverted
PR' (= PR)
F→B'
R→L'
U→D'
L→R'
D→U'
B→F'
#24'

Point Reflection Inverted + Cube Rotations Inverted (3 Axes)								
F – B Axis			R – L Axis			U – D Axis		
PR' CF'	PR' CF2	PR' CF	PR' CR'	PR' CR2	PR' CR	PR' CU'	PR' CU2	PR' CU
F→B'→B'	F→B'→B'	F→B'→B'	F→B'→U'	F→B'→F'	F→B'→D'	F→B'→L'	F→B'→F'	F→B'→R'
R→L'→D'	R→L'→R'	R→L'→U'	R→L'→L'	R→L'→L'	R→L'→L'	R→L'→F'	R→L'→R'	R→L'→B'
U→D'→R'	U→D'→U'	U→D'→L'	U→D'→B'	U→D'→U'	U→D'→F'	U→D'→D'	U→D'→D'	U→D'→D'
L→R'→U'	L→R'→L'	L→R'→D'	L→R'→R'	L→R'→R'	L→R'→R'	L→R'→B'	L→R'→L'	L→R'→F'
D→U'→L'	D→U'→D'	D→U'→R'	D→U'→F'	D→U'→D'	D→U'→B'	D→U'→U'	D→U'→U'	D→U'→U'
B→F'→F'	B→F'→F'	B→F'→F'	B→F'→D'	B→F'→B'	B→F'→U'	B→F'→R'	B→F'→B'	B→F'→L'
#25'	#26'	#27'	#28'	#29'	#30'	#31'	#32'	#33'

Point Reflection Inverted + Cube Rotations Inverted (4 Axes)							
FRD – ULB Axis		FLD – RUB Axis		FRU – LDB Axis		FUL – RDB Axis	
PR' CR CF	PR' CU CF'	PR' CU' CF	PR' CR CF'	PR' CU CF	PR' CR' CF'	PR' CR' CF	PR' CU' CF'
F→B'→L'	F→B'→U'	F→B'→U'	F→B'→R'	F→B'→D'	F→B'→L'	F→B'→R'	F→B'→D'
R→L'→U'	R→L'→B'	R→L'→F'	R→L'→D'	R→L'→B'	R→L'→D'	R→L'→U'	R→L'→F'
U→D'→F'	U→D'→R'	U→D'→L'	U→D'→F'	U→D'→L'	U→D'→B'	U→D'→B'	U→D'→R'
L→R'→D'	L→R'→F'	L→R'→B'	L→R'→U'	L→R'→F'	L→R'→U'	L→R'→D'	L→R'→B'
D→U'→B'	D→U'→L'	D→U'→R'	D→U'→B'	D→U'→R'	D→U'→F'	D→U'→F'	D→U'→L'
B→F'→R'	B→F'→D'	B→F'→D'	B→F'→L'	B→F'→U'	B→F'→R'	B→F'→L'	B→F'→U'
#34'	#35'	#36'	#37'	#38'	#39'	#40'	#41'

Point Reflection Inverted + Cube Rotations Inverted (6 Axes)					
FD – UB Axis	FU – DB Axis	RD – UL Axis	RU – LD Axis	FR – LB Axis	FL – RB Axis
PR' CR CF2	PR' CU2 CR'	PR' CU2 CF'	PR' CU2 CF	PR' CR2 CU	PR' CU' CF2
F→B'→U'	F→B'→D'	F→B'→F'	F→B'→F'	F→B'→L'	F→B'→R'
R→L'→R'	R→L'→R'	R→L'→U'	R→L'→D'	R→L'→B'	R→L'→F'
U→D'→F'	U→D'→B'	U→D'→R'	U→D'→L'	U→D'→U'	U→D'→U'
L→R'→L'	L→R'→L'	L→R'→D'	L→R'→U'	L→R'→F'	L→R'→B'
D→U'→B'	D→U'→F'	D→U'→L'	D→U'→R'	D→U'→D'	D→U'→D'
B→F'→D'	B→F'→U'	B→F'→B'	B→F'→B'	B→F'→R'	B→F'→L'
#42'	#43'	#44'	#45'	#46'	#47'

Mirror Reflections

Table 'Cube Rotations (3 Axes) + Point Reflection', 'R – L Axis', '180°' shows that transformation:

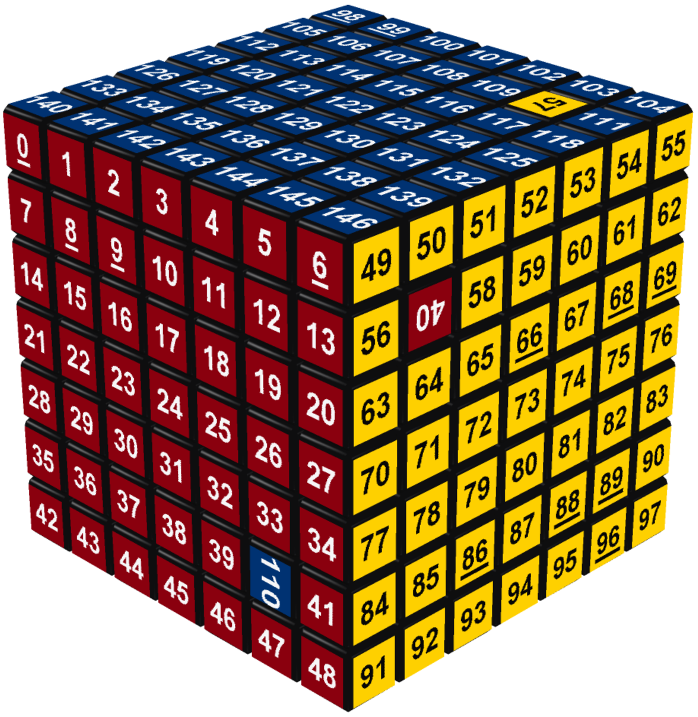
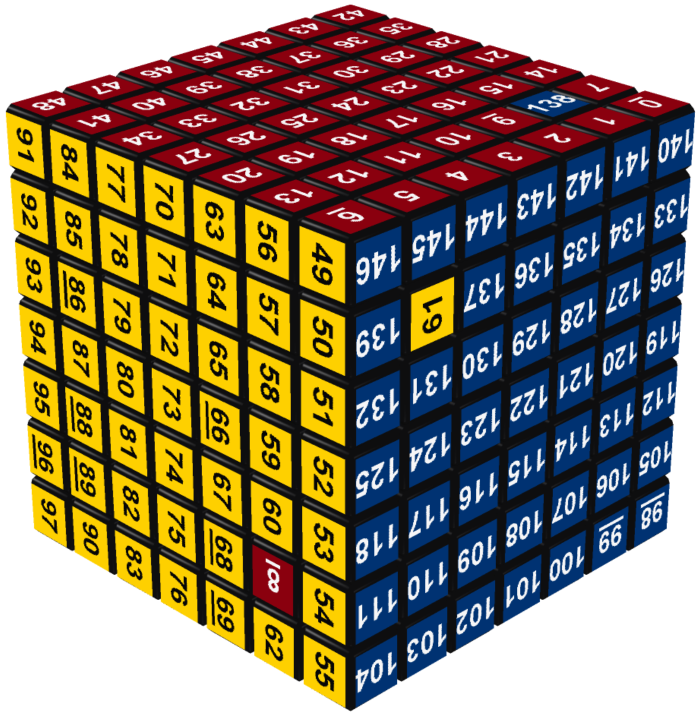
(F, **R**, U, **L**, D, B) → (F', **L'**, U', **R'**, D', B')

is indeed a 'mirror reflection' of the cube from a vertical 'mirror' located on the right-hand side of the cube. Other types of 'mirror reflections' can also be found in the tables.

Cube Groups

In Cube Theory terminology, the 24 cube rotations, plus their combinations, form a group usually called 'C' and the 24 (cube rotations + point reflection), plus the 24 cube rotations, plus their combinations, form a group usually called 'M' (48-fold symmetry). Due to the general closure property of groups, the complete set of 48 transformations is thus sufficient to represent all cube symmetries. For more on how to incorporate symmetry into a cube computer program, see: Calculating Symmetry using Representative Elements.

Cube Rotations – Example

<table>
<tr><th colspan="2">Cube Rotation FRU – LDB Axis +120° (Case #15)</th></tr>
<tr><td></td><td></td></tr>
<tr><td>NU [F2, NU NB NU'] NU'</td><td>NF [R2, NF NL NF'] NF' . CF CR</td></tr>
<tr><td>(57 110 40) – 3-cycle of corner-centers</td><td>(138 8 61) – 3-cycle of corner-centers</td></tr>
<tr><td colspan="2">The 2 algorithms are equivalent because they cycle pieces in the same order and at the same spots.</td></tr>
</table>