

# SuperCube Solver

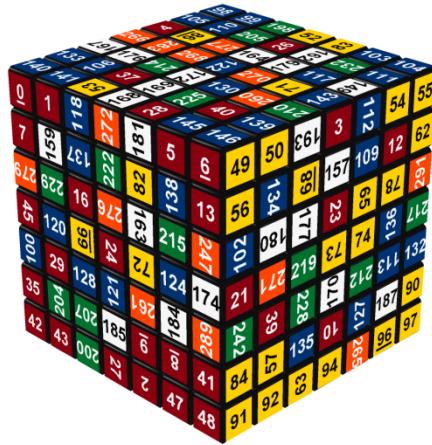
Preliminary Document – Subject To Change



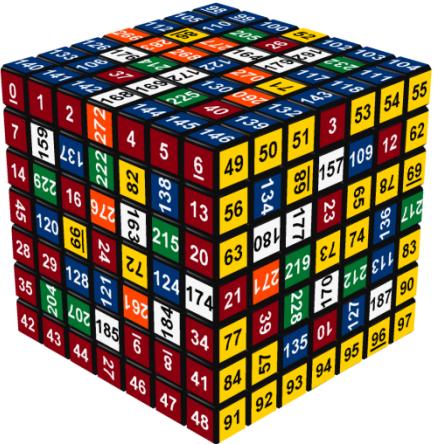
Scrambled Cube



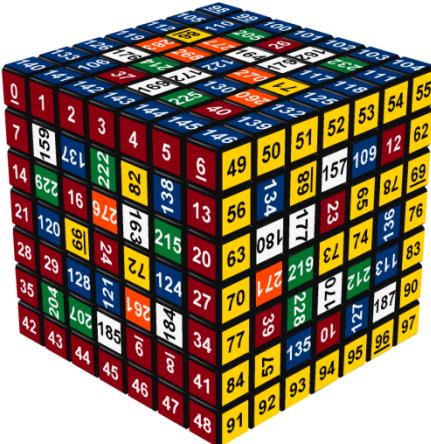
Orbit 0 Solved



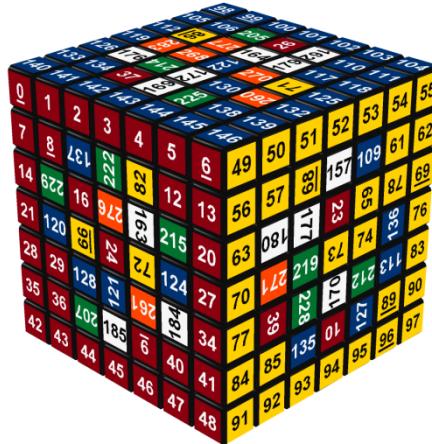
Orbit 1 Solved



Orbit 2 Solved



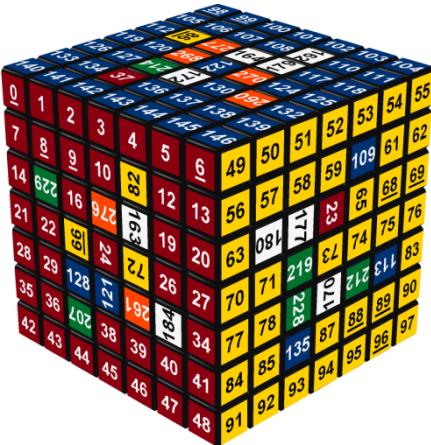
Orbit 3 Solved



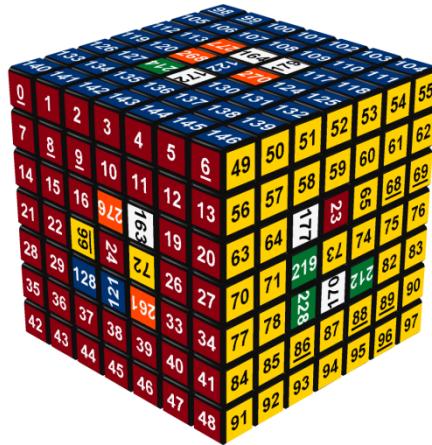
Orbit 5 Solved



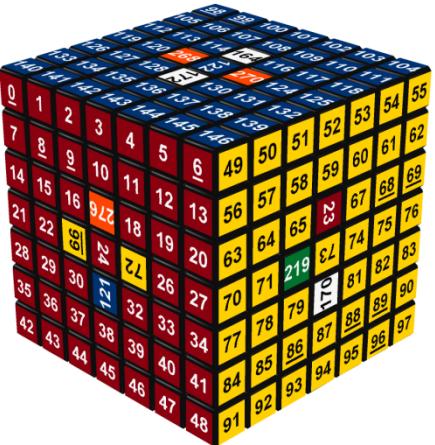
Orbit 6 Solved



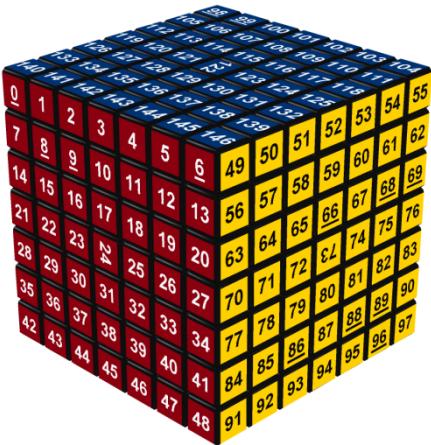
Orbit 7 Solved



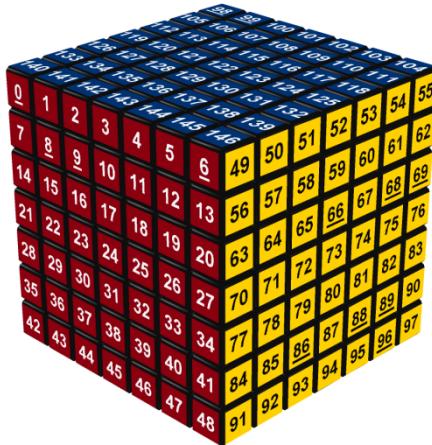
Orbit 9 Solved



Orbit 10 Solved



Orbit 11 Solved



Orbit 12 Solved

# Introduction

## Cube Solvers – Useful Links

<a href="#"><u>Solving the 4x4x4 in 68 turns</u></a>	<a href="#"><u>God's algorithm for 4x4x4 Squares Set</u></a>

# Algorithm Scaling

Algorithms of a given cube orbit can be 'scaled' to give algorithms of other orbits. The piece numbering stays exactly the same for the two orbits, ie. both algorithm index and cycled sticker numbers are exactly the same.

Let's consider the case of a 7x7x7 cube. Notice that algorithm scaling laws for any NxNxN cube can easily be inferred from this latter case.

## Edges

Given an algorithm of orbit 1, replace 'N' with 'N3' to obtain an algorithm of orbit 2:

Orbit 1 algorithm (Cycle sticker numbers: 0 → 1 → 19 → 0): **U NB' U' F' U NB U' F**

Orbit 2 algorithm (Cycle sticker numbers: 0 → 1 → 19 → 0): **U N3B' U' F' U N3B U' F**

Notice that algorithms of orbits 1 and 4 are the same:

Orbit 1 algorithm (Cycle sticker numbers: 0 → 1 → 19 → 0): **U NB' U' F' U NB U' F**

Orbit 4 algorithm (Cycle sticker numbers: 0 → 1 → 19 → 0): **U NB' U' F' U NB U' F**

And similarly for algorithms of orbits 2 and 8:

Orbit 2 algorithm (Cycle sticker numbers: 0 → 1 → 19 → 0): **U N3B' U' F' U N3B U' F**

Orbit 8 algorithm (Cycle sticker numbers: 0 → 1 → 19 → 0): **U N3B' U' F' U N3B U' F**

## Corner-Centers

Given an algorithm of orbit 5, replace 'N' with 'N3' to obtain an algorithm of orbit 10:

Orbit 5 algorithm (Cycle sticker numbers: 0 → 3 → 10 → 0): **F' ND NF ND' F ND NF' ND'**

Orbit 10 algorithm (Cycle sticker numbers: 0 → 3 → 10 → 0): **F' N3D N3F N3D' F N3D N3F' N3D'**

## Middle-Centers

Given an algorithm of orbit 7, replace 'N' with 'N3' to obtain an algorithm of orbit 11:

Orbit 7 algorithm (Cycle sticker numbers: 0 → 3 → 19 → 0): **NU MF NU' F NU MF' NU' F'**

Orbit 11 algorithm (Cycle sticker numbers: 0 → 3 → 19 → 0): **N3U MF N3U' F N3U MF' N3U' F'**

## Edge-Centers

Given an algorithm of orbit 6, swap 'N' with 'N3' to obtain an algorithm of orbit 9:

Orbit 6 algorithm (Cycle sticker numbers: 0 → 4 → 9 → 0): **N3L' U' N3L NU N3L' U N3L NU'**

Orbit 9 algorithm (Cycle sticker numbers: 0 → 4 → 9 → 0): **NL' U' NL N3U NL' U NL N3U'**

		<b>N</b>	<b>N3</b>	<b>M</b>	<b>N3</b>	<b>N</b>				<b>N</b>	<b>N3</b>	<b>M</b>	<b>N3</b>	<b>N</b>		
<b>N</b>	0	1	2	3	8	4	0		0	0	0	0	1	1	1	
<b>N3</b>	4	5	6	7	9	5	1		0	0	0	0	1	1	1	
<b>M</b>				12	11	7	3					0	1	1	1	
	7x								7x							
Orbit Numbering – First Blocks								Sticker Numbering – First Blocks								

# Super Cube Orbit Numbering

The orbits of the 7x7x7 Super Cube are numbered from 0 (corners) up to 12 (true centers). Edge orbits marked in grey are not solved directly because this is already done by solving the symmetrical edges, ie. orbit 1 solves orbit 4 and orbit 2 solves orbit 8. Notice that the orbit numbering of the Super Cube differs from that of the Orbit Cube.

0	1				0	1				0	1	2			
		2			2	3				3	4	5			
3x					4x					5x					
0	1	2			0	1	2	3		0	1	2	3		
3	4	5			4	5	6	7		4	5	6	7		
6	7	8			8	9	10	11		8	9	10	11		
							12			12	13	14	15		
6x					7x					8x					
0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	5
5	6	7	8	9	5	6	7	8	9	6	7	8	9	10	11
10	11	12	13	14	10	11	12	13	14	12	13	14	15	16	17
15	16	17	18	19	15	16	17	18	19	18	19	20	21	22	23
			20		20	21	22	23	24	24	25	26	27	28	29
9x					10x										30
0	1	2	3	4	5	0	1	2	3	4	5	6	0	1	2
6	7	8	9	10	11	7	8	9	10	11	12	13	7	8	9
12	13	14	15	16	17	14	15	16	17	18	19	20	14	15	16
18	19	20	21	22	23	21	22	23	24	25	26	27	21	22	23
24	25	26	27	28	29	28	29	30	31	32	33	34	28	29	30
30	31	32	33	34	35	35	36	37	38	39	40	41	35	36	37
12x						13x				42	14x				

Orbit Numbering – Super Cube 7x vs Orbit Cube 7x

Super Cube 7x	0	1	2	3	4	5	6	7	8	9	10	11	12
Orbit Cube 7x	03	06	09	12	02	05	08	11	01	04	07	10	00

# Orbit Cube Orbit Numbering

The orbits of the 7x7x7 Orbit Cube are numbered from 00 (true centers) up to 12 (midges). Edge orbits marked in grey are not solved directly because this is already done by solving the symmetrical edges, ie. orbit 06 solves orbit 02 and orbit 09 solves orbit 01. Notice that the orbit numbering of the Orbit Cube differs from that of the Super Cube.

01	02				02	04				02	04	06							
	00				01	03				01	03	05							
												00							
3x					4x					5x									
03	06	09			03	06	09	12		04	08	12	16						
02	05	08			02	05	08	11		03	07	11	15						
01	04	07			01	04	07	10		02	06	10	14						
								00		01	05	09	13						
6x					7x					8x									
04	08	12	16	20		05	10	15	20	25	05	10	15	20	25	30			
03	07	11	15	19		04	09	14	19	24	04	09	14	19	24	29			
02	06	10	14	18		03	08	13	18	23	03	08	13	18	23	28			
01	05	09	13	17		02	07	12	17	22	02	07	12	17	22	27			
			00			01	06	11	16	21	01	06	11	16	21	26			
9x					10x					11x					00				
06	12	18	24	30	36	06	12	18	24	30	36	42	07	14	21	28	35	42	49
05	11	17	23	29	35	05	11	17	23	29	35	41	06	13	20	27	34	41	48
04	10	16	22	28	34	04	10	16	22	28	34	40	05	12	19	26	33	40	47
03	09	15	21	27	33	03	09	15	21	27	33	39	04	11	18	25	32	39	46
02	08	14	20	26	32	02	08	14	20	26	32	38	03	10	17	24	31	38	45
01	07	13	19	25	31	01	07	13	19	25	31	37	02	09	16	23	30	37	44
12x					13x					00	01	08	15	22	29	36	43		
14x																			

Orbit Numbering – Orbit Cube 7x vs Super Cube 7x

Orbit Cube 7x	00	01	02	03	04	05	06	07	08	09	10	11	12
Super Cube 7x	12	8	4	0	9	5	1	10	6	2	11	7	3

# Odd-Cycle Decomposition

## (7c) = (3c) (5c) – Left

$(\underline{1} \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) = (\underline{1} \ 2 \ 3) (\underline{1} \ 4 \ 5 \ 6 \ 7)$   
 $(\underline{2} \ 3 \ 4 \ 5 \ 6 \ 7 \ 1) = (\underline{2} \ 3 \ 4) (\underline{2} \ 5 \ 6 \ 7 \ 1)$   
 $(\underline{3} \ 4 \ 5 \ 6 \ 7 \ 1 \ 2) = (\underline{3} \ 4 \ 5) (\underline{3} \ 6 \ 7 \ 1 \ 2)$   
 $(\underline{4} \ 5 \ 6 \ 7 \ 1 \ 2 \ 3) = (\underline{4} \ 5 \ 6) (\underline{4} \ 7 \ 1 \ 2 \ 3)$   
 $(\underline{5} \ 6 \ 7 \ 1 \ 2 \ 3 \ 4) = (\underline{5} \ 6 \ 7) (\underline{5} \ 1 \ 2 \ 3 \ 4)$   
 $(\underline{6} \ 7 \ 1 \ 2 \ 3 \ 4 \ 5) = (\underline{6} \ 7 \ 1) (\underline{6} \ 2 \ 3 \ 4 \ 5)$   
 $(\underline{7} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6) = (\underline{7} \ 1 \ 2) (\underline{7} \ 3 \ 4 \ 5 \ 6)$

## (7c) = (5c) (3c) – Right

$(1 \ 2 \ 3 \ 4 \ 5 \ 6 \ \underline{7}) = (1 \ 2 \ 3 \ 4 \ \underline{7}) (\underline{5} \ \underline{6} \ \underline{7})$   
 $(2 \ 3 \ 4 \ 5 \ 6 \ \underline{7} \ 1) = (2 \ 3 \ 4 \ 5 \ \underline{1}) (\underline{6} \ \underline{7} \ \underline{1})$   
 $(3 \ 4 \ 5 \ 6 \ \underline{7} \ 1 \ \underline{2}) = (3 \ 4 \ 5 \ 6 \ \underline{2}) (\underline{7} \ \underline{1} \ \underline{2})$   
 $(4 \ 5 \ 6 \ \underline{7} \ 1 \ 2 \ \underline{3}) = (4 \ 5 \ 6 \ \underline{7} \ \underline{3}) (\underline{1} \ \underline{2} \ \underline{3})$   
 $(5 \ 6 \ 7 \ 1 \ 2 \ 3 \ \underline{4}) = (5 \ 6 \ 7 \ 1 \ \underline{4}) (\underline{2} \ \underline{3} \ \underline{4})$   
 $(6 \ 7 \ 1 \ 2 \ 3 \ 4 \ \underline{5}) = (6 \ 7 \ 1 \ 2 \ \underline{5}) (\underline{3} \ \underline{4} \ \underline{5})$   
 $(7 \ 1 \ 2 \ 3 \ 4 \ 5 \ \underline{6}) = (7 \ 1 \ 2 \ 3 \ \underline{6}) (\underline{4} \ \underline{5} \ \underline{6})$

## (9c) = (5c) (5c) – Left

$(\underline{1} \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9) = (\underline{1} \ 2 \ 3 \ 4 \ 5) (\underline{1} \ 6 \ 7 \ 8 \ 9)$   
 $(\underline{2} \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 1) = (\underline{2} \ 3 \ 4 \ 5 \ 6) (\underline{2} \ 7 \ 8 \ 9 \ 1)$   
 $(\underline{3} \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 1 \ 2) = (\underline{3} \ 4 \ 5 \ 6 \ 7) (\underline{3} \ 8 \ 9 \ 1 \ 2)$   
 $(\underline{4} \ 5 \ 6 \ 7 \ 8 \ 9 \ 1 \ 2 \ 3) = (\underline{4} \ 5 \ 6 \ 7 \ 8) (\underline{4} \ 9 \ 1 \ 2 \ 3)$   
 $(\underline{5} \ 6 \ 7 \ 8 \ 9 \ 1 \ 2 \ 3 \ 4) = (\underline{5} \ 6 \ 7 \ 8 \ 9) (\underline{5} \ 1 \ 2 \ 3 \ 4)$   
 $(\underline{6} \ 7 \ 8 \ 9 \ 1 \ 2 \ 3 \ 4 \ 5) = (\underline{6} \ 7 \ 8 \ 9 \ 1) (\underline{6} \ 2 \ 3 \ 4 \ 5)$   
 $(\underline{7} \ 8 \ 9 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6) = (\underline{7} \ 8 \ 9 \ 1 \ 2) (\underline{7} \ 3 \ 4 \ 5 \ 6)$   
 $(\underline{8} \ 9 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) = (\underline{8} \ 9 \ 1 \ 2 \ 3) (\underline{8} \ 4 \ 5 \ 6 \ 7)$   
 $(\underline{9} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) = (\underline{9} \ 1 \ 2 \ 3 \ 4) (\underline{9} \ 5 \ 6 \ 7 \ 8)$

## (9c) = (5c) (5c) – Right

$(1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ \underline{9}) = (1 \ 2 \ 3 \ 4 \ \underline{9}) (\underline{5} \ \underline{6} \ \underline{7} \ \underline{8} \ \underline{9})$   
 $(2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ \underline{9} \ \underline{1}) = (2 \ 3 \ 4 \ 5 \ \underline{1}) (\underline{6} \ \underline{7} \ \underline{8} \ \underline{9} \ \underline{1})$   
 $(3 \ 4 \ 5 \ 6 \ 7 \ 8 \ \underline{9} \ 1 \ \underline{2}) = (3 \ 4 \ 5 \ 6 \ \underline{2}) (\underline{7} \ \underline{8} \ \underline{9} \ \underline{1} \ \underline{2})$   
 $(4 \ 5 \ 6 \ 7 \ 8 \ \underline{9} \ 1 \ 2 \ \underline{3}) = (4 \ 5 \ 6 \ \underline{7} \ \underline{3}) (\underline{8} \ \underline{9} \ \underline{1} \ \underline{2} \ \underline{3})$   
 $(5 \ 6 \ 7 \ 8 \ \underline{9} \ 1 \ 2 \ 3 \ \underline{4}) = (5 \ 6 \ 7 \ \underline{8} \ \underline{4}) (\underline{9} \ \underline{1} \ \underline{2} \ \underline{3} \ \underline{4})$   
 $(6 \ 7 \ 8 \ \underline{9} \ 1 \ 2 \ 3 \ 4 \ \underline{5}) = (6 \ 7 \ 8 \ \underline{9} \ \underline{5}) (\underline{1} \ \underline{2} \ \underline{3} \ \underline{4} \ \underline{5})$   
 $(7 \ 8 \ 9 \ 1 \ 2 \ 3 \ 4 \ \underline{5} \ \underline{6}) = (7 \ 8 \ 9 \ 1 \ \underline{6}) (\underline{2} \ \underline{3} \ \underline{4} \ \underline{5} \ \underline{6})$   
 $(8 \ 9 \ 1 \ 2 \ 3 \ 4 \ 5 \ \underline{6} \ \underline{7}) = (8 \ 9 \ 1 \ 2 \ \underline{7}) (\underline{3} \ \underline{4} \ \underline{5} \ \underline{6} \ \underline{7})$   
 $(9 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ \underline{7} \ \underline{8}) = (9 \ 1 \ 2 \ 3 \ \underline{8}) (\underline{4} \ \underline{5} \ \underline{6} \ \underline{7} \ \underline{8})$

## (11c) = (3c) (5c) (5c) – Left

$(\underline{1} \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11) = (\underline{1} \ 2 \ 3) (\underline{1} \ 4 \ 5 \ 6 \ 7) (\underline{1} \ 8 \ 9 \ 10 \ 11)$   
 $(\underline{2} \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 1) = (\underline{2} \ 3 \ 4) (\underline{2} \ 5 \ 6 \ 7 \ 8) (\underline{2} \ 9 \ 10 \ 11 \ 1)$   
 $(\underline{3} \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 1 \ 2) = (\underline{3} \ 4 \ 5) (\underline{3} \ 6 \ 7 \ 8 \ 9) (\underline{3} \ 10 \ 11 \ 1 \ 2)$   
 $(\underline{4} \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 1 \ 2 \ 3) = (\underline{4} \ 5 \ 6) (\underline{4} \ 7 \ 8 \ 9 \ 10) (\underline{4} \ 11 \ 1 \ 2 \ 3)$   
 $(\underline{5} \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 1 \ 2 \ 3 \ 4) = (\underline{5} \ 6 \ 7) (\underline{5} \ 8 \ 9 \ 10 \ 11) (\underline{5} \ 1 \ 2 \ 3 \ 4)$   
 $(\underline{6} \ 7 \ 8 \ 9 \ 10 \ 11 \ 1 \ 2 \ 3 \ 4 \ 5) = (\underline{6} \ 7 \ 8) (\underline{6} \ 9 \ 10 \ 11 \ 1) (\underline{6} \ 2 \ 3 \ 4 \ 5)$   
 $(\underline{7} \ 8 \ 9 \ 10 \ 11 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6) = (\underline{7} \ 8 \ 9) (\underline{7} \ 10 \ 11 \ 1 \ 2) (\underline{7} \ 3 \ 4 \ 5 \ 6)$   
 $(\underline{8} \ 9 \ 10 \ 11 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) = (\underline{8} \ 9 \ 10) (\underline{8} \ 11 \ 1 \ 2 \ 3) (\underline{8} \ 4 \ 5 \ 6 \ 7)$   
 $(\underline{9} \ 10 \ 11 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) = (\underline{9} \ 10 \ 11) (\underline{9} \ 1 \ 2 \ 3 \ 4) (\underline{9} \ 5 \ 6 \ 7 \ 8)$   
 $(\underline{10} \ 11 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9) = (\underline{10} \ 11 \ 1) (\underline{10} \ 2 \ 3 \ 4 \ 5) (\underline{10} \ 6 \ 7 \ 8 \ 9)$   
 $(\underline{11} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10) = (\underline{11} \ 1 \ 2) (\underline{11} \ 3 \ 4 \ 5 \ 6) (\underline{11} \ 7 \ 8 \ 9 \ 10)$

## (11c) = (5c) (5c) (3c) – Right

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11}) = (1\ 2\ 3\ 4\ \underline{11}) (5\ 6\ 7\ 8\ \underline{11}) (9\ 10\ \underline{11})$   
 $(2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 11}) = (2\ 3\ 4\ 5\ \underline{1}) (6\ 7\ 8\ 9\ \underline{1}) (10\ 11\ \underline{11})$   
 $(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 11\ 12}) = (3\ 4\ 5\ 6\ \underline{2}) (7\ 8\ 9\ 10\ \underline{2}) (11\ 1\ \underline{2})$   
 $(4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 11\ 12\ 13}) = (4\ 5\ 6\ 7\ \underline{3}) (8\ 9\ 10\ \underline{11\ 3}) (1\ 2\ \underline{3})$   
 $(5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 11\ 12\ 13\ 4}) = (5\ 6\ 7\ 8\ \underline{4}) (9\ 10\ \underline{11\ 1\ 4}) (2\ 3\ \underline{4})$   
 $(6\ 7\ 8\ 9\ 10\ \underline{11\ 11\ 12\ 13\ 4\ 5}) = (6\ 7\ 8\ 9\ \underline{5}) (10\ 11\ \underline{12\ 5}) (3\ 4\ \underline{5})$   
 $(7\ 8\ 9\ 10\ \underline{11\ 11\ 12\ 13\ 4\ 5\ 6}) = (7\ 8\ 9\ 10\ \underline{6}) (11\ 1\ \underline{2\ 3\ 6}) (4\ 5\ \underline{6})$   
 $(8\ 9\ 10\ \underline{11\ 11\ 12\ 13\ 4\ 5\ 6\ 7}) = (8\ 9\ 10\ \underline{11\ 7}) (1\ 2\ 3\ 4\ \underline{7}) (5\ 6\ \underline{7})$   
 $(9\ 10\ \underline{11\ 12\ 13\ 4\ 5\ 6\ 7\ 8}) = (9\ 10\ \underline{11\ 1\ 8}) (2\ 3\ 4\ 5\ \underline{8}) (6\ 7\ \underline{8})$   
 $(10\ \underline{11\ 12\ 13\ 4\ 5\ 6\ 7\ 8\ 9}) = (10\ 11\ \underline{12\ 9}) (3\ 4\ 5\ 6\ \underline{9}) (7\ 8\ \underline{9})$   
 $(11\ 1\ \underline{2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10}) = (11\ 1\ \underline{2\ 3\ 10}) (4\ 5\ 6\ 7\ \underline{10}) (8\ 9\ \underline{10})$

### **(13c) = (5c) (5c) (5c) – Left**

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 12\ 13}) = (\underline{1\ 2\ 3\ 4\ 5}) (\underline{1\ 6\ 7\ 8\ 9}) (\underline{1\ 10\ 11\ 12\ 13})$   
 $(\underline{2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 1}) = (\underline{2\ 3\ 4\ 5\ 6}) (\underline{2\ 7\ 8\ 9\ 10}) (\underline{2\ 11\ 12\ 13\ 1})$   
 $(\underline{3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 1\ 2}) = (\underline{3\ 4\ 5\ 6\ 7}) (\underline{3\ 8\ 9\ 10\ 11}) (\underline{3\ 12\ 13\ 1\ 2})$   
 $(\underline{4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 1\ 2\ 3}) = (\underline{4\ 5\ 6\ 7\ 8}) (\underline{4\ 9\ 10\ 11\ 12}) (\underline{4\ 13\ 1\ 2\ 3})$   
 $(\underline{5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 1\ 2\ 3\ 4}) = (\underline{5\ 6\ 7\ 8\ 9}) (\underline{5\ 10\ 11\ 12\ 13}) (\underline{5\ 1\ 2\ 3\ 4})$   
 $(\underline{6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 1\ 2\ 3\ 4\ 5}) = (\underline{6\ 7\ 8\ 9\ 10}) (\underline{6\ 11\ 12\ 13\ 1}) (\underline{6\ 2\ 3\ 4\ 5})$   
 $(\underline{7\ 8\ 9\ 10\ 11\ 12\ 13\ 1\ 2\ 3\ 4\ 5\ 6}) = (\underline{7\ 8\ 9\ 10\ 11}) (\underline{7\ 12\ 13\ 1\ 2}) (\underline{7\ 3\ 4\ 5\ 6})$   
 $(\underline{8\ 9\ 10\ 11\ 12\ 13\ 1\ 2\ 3\ 4\ 5\ 6\ 7}) = (\underline{8\ 9\ 10\ 11\ 12}) (\underline{8\ 13\ 1\ 2\ 3}) (\underline{8\ 4\ 5\ 6\ 7})$   
 $(\underline{9\ 10\ 11\ 12\ 13\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8}) = (\underline{9\ 10\ 11\ 12\ 13}) (\underline{9\ 1\ 2\ 3\ 4}) (\underline{9\ 5\ 6\ 7\ 8})$   
 $(\underline{10\ 11\ 12\ 13\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9}) = (\underline{10\ 11\ 12\ 13\ 1}) (\underline{10\ 2\ 3\ 4\ 5}) (\underline{10\ 6\ 7\ 8\ 9})$   
 $(\underline{11\ 12\ 13\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10}) = (\underline{11\ 12\ 13\ 1\ 2}) (\underline{11\ 3\ 4\ 5\ 6}) (\underline{11\ 7\ 8\ 9\ 10})$   
 $(\underline{12\ 13\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11}) = (\underline{12\ 13\ 1\ 2\ 3}) (\underline{12\ 4\ 5\ 6\ 7}) (\underline{12\ 8\ 9\ 10\ 11})$   
 $(\underline{13\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12}) = (\underline{13\ 1\ 2\ 3\ 4}) (\underline{13\ 5\ 6\ 7\ 8}) (\underline{13\ 9\ 10\ 11\ 12})$

### **(13c) = (5c) (5c) (5c) – Right**

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 12\ 13}) = (1\ 2\ 3\ 4\ \underline{13}) (5\ 6\ 7\ 8\ \underline{13}) (9\ 10\ \underline{11\ 12\ 13})$   
 $(2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 12\ 13\ 1}) = (2\ 3\ 4\ 5\ \underline{1}) (6\ 7\ 8\ 9\ \underline{1}) (10\ 11\ \underline{12\ 13\ 1})$   
 $(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 12\ 13\ 1\ 2}) = (3\ 4\ 5\ 6\ \underline{2}) (7\ 8\ 9\ 10\ \underline{2}) (11\ 12\ \underline{13\ 1\ 2})$   
 $(4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 12\ 13\ 1\ 2\ 3}) = (4\ 5\ 6\ 7\ \underline{3}) (8\ 9\ 10\ \underline{11\ 3}) (12\ 13\ \underline{1\ 2\ 3})$   
 $(5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 12\ 13\ 1\ 2\ 3\ 4}) = (5\ 6\ 7\ 8\ \underline{4}) (9\ 10\ \underline{11\ 12\ 4}) (13\ 1\ \underline{2\ 3\ 4})$   
 $(6\ 7\ 8\ 9\ 10\ \underline{11\ 12\ 13\ 1\ 2\ 3\ 4\ 5}) = (6\ 7\ 8\ 9\ \underline{5}) (10\ 11\ \underline{12\ 13\ 5}) (1\ 2\ 3\ \underline{4\ 5})$   
 $(7\ 8\ 9\ 10\ \underline{11\ 12\ 13\ 1\ 2\ 3\ 4\ 5\ 6}) = (7\ 8\ 9\ 10\ \underline{6}) (11\ 12\ \underline{13\ 1\ 6}) (2\ 3\ 4\ \underline{5\ 6})$   
 $(8\ 9\ 10\ \underline{11\ 12\ 13\ 1\ 2\ 3\ 4\ 5\ 6\ 7}) = (8\ 9\ 10\ \underline{11\ 7}) (12\ 13\ \underline{1\ 2\ 7}) (3\ 4\ 5\ \underline{6\ 7})$   
 $(9\ 10\ \underline{11\ 12\ 13\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8}) = (9\ 10\ \underline{11\ 12\ 8}) (13\ 1\ \underline{2\ 3\ 8}) (4\ 5\ 6\ \underline{7\ 8})$   
 $(10\ \underline{11\ 12\ 13\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9}) = (10\ 11\ \underline{12\ 13\ 9}) (1\ 2\ 3\ \underline{4\ 9}) (5\ 6\ 7\ \underline{8\ 9})$   
 $(11\ \underline{12\ 13\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10}) = (11\ 12\ \underline{13\ 1\ 10}) (2\ 3\ 4\ \underline{5\ 10}) (6\ 7\ 8\ \underline{9\ 10})$   
 $(12\ \underline{13\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11}) = (12\ 13\ \underline{1\ 2\ 11}) (3\ 4\ 5\ 6\ \underline{11}) (7\ 8\ 9\ \underline{10\ 11})$   
 $(13\ \underline{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12}) = (13\ 1\ \underline{2\ 3\ 12}) (4\ 5\ 6\ 7\ \underline{12}) (8\ 9\ \underline{10\ 11\ 12})$

### **(15c) = (3c) (5c) (5c) (5c) – Left**

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11\ 12\ 13\ 14\ 15}) = (\underline{1\ 2\ 3}) (\underline{1\ 4\ 5\ 6\ 7}) (\underline{1\ 8\ 9\ 10\ 11}) (\underline{1\ 12\ 13\ 14\ 15})$   
 $(\underline{2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1}) = (\underline{2\ 3\ 4}) (\underline{2\ 5\ 6\ 7\ 8}) (\underline{2\ 9\ 10\ 11\ 12}) (\underline{2\ 13\ 14\ 15\ 1})$   
 $(\underline{3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2}) = (\underline{3\ 4\ 5}) (\underline{3\ 6\ 7\ 8\ 9}) (\underline{3\ 10\ 11\ 12\ 13}) (\underline{3\ 14\ 15\ 1\ 2})$   
 $(\underline{4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3}) = (\underline{4\ 5\ 6}) (\underline{4\ 7\ 8\ 9\ 10}) (\underline{4\ 11\ 12\ 13\ 14}) (\underline{4\ 15\ 1\ 2\ 3})$   
 $(\underline{5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4}) = (\underline{5\ 6\ 7}) (\underline{5\ 8\ 9\ 10\ 11}) (\underline{5\ 12\ 13\ 14\ 15}) (\underline{5\ 1\ 2\ 3\ 4})$   
 $(\underline{6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5}) = (\underline{6\ 7\ 8}) (\underline{6\ 9\ 10\ 11\ 12}) (\underline{6\ 13\ 14\ 15\ 1}) (\underline{6\ 2\ 3\ 4\ 5})$   
 $(\underline{7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6}) = (\underline{7\ 8\ 9}) (\underline{7\ 10\ 11\ 12\ 13}) (\underline{7\ 14\ 15\ 1\ 2}) (\underline{7\ 3\ 4\ 5\ 6})$   
 $(\underline{8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7}) = (\underline{8\ 9\ 10}) (\underline{8\ 11\ 12\ 13\ 14}) (\underline{8\ 15\ 1\ 2\ 3}) (\underline{8\ 4\ 5\ 6\ 7})$   
 $(\underline{9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8}) = (\underline{9\ 10\ 11}) (\underline{9\ 12\ 13\ 14\ 15}) (\underline{9\ 1\ 2\ 3\ 4}) (\underline{9\ 5\ 6\ 7\ 8})$   
 $(\underline{10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9}) = (\underline{10\ 11\ 12}) (\underline{10\ 13\ 14\ 15\ 1}) (\underline{10\ 2\ 3\ 4\ 5}) (\underline{10\ 6\ 7\ 8\ 9})$   
 $(\underline{11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10}) = (\underline{11\ 12\ 13}) (\underline{11\ 14\ 15\ 1\ 2}) (\underline{11\ 3\ 4\ 5\ 6}) (\underline{11\ 7\ 8\ 9\ 10})$   
 $(\underline{12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11}) = (\underline{12\ 13\ 14}) (\underline{12\ 15\ 1\ 2\ 3}) (\underline{12\ 4\ 5\ 6\ 7}) (\underline{12\ 8\ 9\ 10\ 11})$   
 $(\underline{13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12}) = (\underline{13\ 14\ 15}) (\underline{13\ 1\ 2\ 3\ 4}) (\underline{13\ 5\ 6\ 7\ 8}) (\underline{13\ 9\ 10\ 11\ 12})$   
 $(\underline{14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13}) = (\underline{14\ 15\ 1}) (\underline{14\ 2\ 3\ 4\ 5}) (\underline{14\ 6\ 7\ 8\ 9}) (\underline{14\ 10\ 11\ 12\ 13})$   
 $(\underline{15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14}) = (\underline{15\ 1\ 2}) (\underline{15\ 3\ 4\ 5\ 6}) (\underline{15\ 7\ 8\ 9\ 10}) (\underline{15\ 11\ 12\ 13\ 14})$

**(15c) = (5c) (5c) (5c) (3c) – Right**

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15) = (1\ 2\ 3\ 4\ 15)(5\ 6\ 7\ 8\ 15)(9\ 10\ 11\ 12\ 15)(13\ 14\ 15)$   
 $(2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1) = (2\ 3\ 4\ 5\ 1)(6\ 7\ 8\ 9\ 1)(10\ 11\ 12\ 13\ 1)(14\ 15\ 1)$   
 $(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2) = (3\ 4\ 5\ 6\ 2)(7\ 8\ 9\ 10\ 2)(11\ 12\ 13\ 14\ 2)(15\ 1\ 2)$   
 $(4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3) = (4\ 5\ 6\ 7\ 3)(8\ 9\ 10\ 11\ 3)(12\ 13\ 14\ 15\ 3)(1\ 2\ 3)$   
 $(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4) = (5\ 6\ 7\ 8\ 4)(9\ 10\ 11\ 12\ 4)(13\ 14\ 15\ 1\ 4)(2\ 3\ 4)$   
 $(6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5) = (6\ 7\ 8\ 9\ 5)(10\ 11\ 12\ 13\ 5)(14\ 15\ 1\ 2\ 5)(3\ 4\ 5)$   
 $(7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6) = (7\ 8\ 9\ 10\ 6)(11\ 12\ 13\ 14\ 6)(15\ 1\ 2\ 3\ 6)(4\ 5\ 6)$   
 $(8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7) = (8\ 9\ 10\ 11\ 7)(12\ 13\ 14\ 15\ 7)(1\ 2\ 3\ 4\ 7)(5\ 6\ 7)$   
 $(9\ 10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8) = (9\ 10\ 11\ 12\ 8)(13\ 14\ 15\ 1\ 8)(2\ 3\ 4\ 5\ 8)(6\ 7\ 8)$   
 $(10\ 11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9) = (10\ 11\ 12\ 13\ 9)(14\ 15\ 1\ 2\ 9)(3\ 4\ 5\ 6\ 9)(7\ 8\ 9)$   
 $(11\ 12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10) = (11\ 12\ 13\ 14\ 10)(15\ 1\ 2\ 3\ 10)(4\ 5\ 6\ 7\ 10)(8\ 9\ 10)$   
 $(12\ 13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11) = (12\ 13\ 14\ 15\ 11)(1\ 2\ 3\ 4\ 11)(5\ 6\ 7\ 8\ 11)(9\ 10\ 11)$   
 $(13\ 14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12) = (13\ 14\ 15\ 1\ 12)(2\ 3\ 4\ 5\ 12)(6\ 7\ 8\ 9\ 12)(10\ 11\ 12)$   
 $(14\ 15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13) = (14\ 15\ 1\ 2\ 13)(3\ 4\ 5\ 6\ 13)(7\ 8\ 9\ 10\ 13)(11\ 12\ 13)$   
 $(15\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14) = (15\ 1\ 2\ 3\ 14)(4\ 5\ 6\ 7\ 14)(8\ 9\ 10\ 11\ 14)(12\ 13\ 14)$

**(17c) = (5c) (5c) (5c) (5c) – Left**

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17) = (1\ 2\ 3\ 4\ 5)(1\ 6\ 7\ 8\ 9)(1\ 10\ 11\ 12\ 13)(1\ 14\ 15\ 16\ 17)$   
 $(2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1) = (2\ 3\ 4\ 5\ 6)(2\ 7\ 8\ 9\ 10)(2\ 11\ 12\ 13\ 14)(2\ 15\ 16\ 17\ 1)$   
 $(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2) = (3\ 4\ 5\ 6\ 7)(3\ 8\ 9\ 10\ 11)(3\ 12\ 13\ 14\ 15)(3\ 16\ 17\ 1\ 2)$   
 $(4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3) = (4\ 5\ 6\ 7\ 8)(4\ 9\ 10\ 11\ 12)(4\ 13\ 14\ 15\ 16)(4\ 17\ 1\ 2\ 3)$   
 $(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4) = (5\ 6\ 7\ 8\ 9)(5\ 10\ 11\ 12\ 13)(5\ 14\ 15\ 16\ 17)(5\ 1\ 2\ 3\ 4)$   
 $(6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5) = (6\ 7\ 8\ 9\ 10)(6\ 11\ 12\ 13\ 14)(6\ 15\ 16\ 17\ 1)(6\ 2\ 3\ 4\ 5)$   
 $(7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6) = (7\ 8\ 9\ 10\ 11)(7\ 12\ 13\ 14\ 15)(7\ 16\ 17\ 1\ 2)(7\ 3\ 4\ 5\ 6)$   
 $(8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7) = (8\ 9\ 10\ 11\ 12)(8\ 13\ 14\ 15\ 16)(8\ 17\ 1\ 2\ 3)(8\ 4\ 5\ 6\ 7)$   
 $(9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8) = (9\ 10\ 11\ 12\ 13)(9\ 14\ 15\ 16\ 17)(9\ 1\ 2\ 3\ 4)(9\ 5\ 6\ 7\ 8)$   
 $(10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9) = (10\ 11\ 12\ 13\ 14)(10\ 15\ 16\ 17\ 1)(10\ 2\ 3\ 4\ 5)(10\ 6\ 7\ 8\ 9)$   
 $(11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10) = (11\ 12\ 13\ 14\ 15)(11\ 16\ 17\ 1\ 2)(11\ 3\ 4\ 5\ 6)(11\ 7\ 8\ 9\ 10)$   
 $(12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11) = (12\ 13\ 14\ 15\ 16)(12\ 17\ 1\ 2\ 3)(12\ 4\ 5\ 6\ 7)(12\ 8\ 9\ 10\ 11)$   
 $(13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12) = (13\ 14\ 15\ 16\ 17)(13\ 1\ 2\ 3\ 4)(13\ 5\ 6\ 7\ 8)(13\ 9\ 10\ 11\ 12)$   
 $(14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13) = (14\ 15\ 16\ 17\ 1)(14\ 2\ 3\ 4\ 5)(14\ 6\ 7\ 8\ 9)(14\ 10\ 11\ 12\ 13)$   
 $(15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14) = (15\ 16\ 17\ 1\ 2)(15\ 3\ 4\ 5\ 6)(15\ 7\ 8\ 9\ 10)(15\ 11\ 12\ 13\ 14)$   
 $(16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15) = (16\ 17\ 1\ 2\ 3)(16\ 4\ 5\ 6\ 7)(16\ 8\ 9\ 10\ 11)(16\ 12\ 13\ 14\ 15)$   
 $(17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16) = (17\ 1\ 2\ 3\ 4)(17\ 5\ 6\ 7\ 8)(17\ 9\ 10\ 11\ 12)(17\ 13\ 14\ 15\ 16)$

**(17c) = (5c) (5c) (5c) (5c) – Right**

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1) = (1\ 2\ 3\ 4\ 17)(5\ 6\ 7\ 8\ 17)(9\ 10\ 11\ 12\ 17)(13\ 14\ 15\ 16\ 17)$   
 $(2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1) = (2\ 3\ 4\ 5\ 1)(6\ 7\ 8\ 9\ 1)(10\ 11\ 12\ 13\ 1)(14\ 15\ 16\ 17\ 1)$   
 $(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2) = (3\ 4\ 5\ 6\ 2)(7\ 8\ 9\ 10\ 2)(11\ 12\ 13\ 14\ 2)(15\ 16\ 17\ 1\ 2)$   
 $(4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3) = (4\ 5\ 6\ 7\ 3)(8\ 9\ 10\ 11\ 3)(12\ 13\ 14\ 15\ 3)(16\ 17\ 1\ 2\ 3)$   
 $(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4) = (5\ 6\ 7\ 8\ 4)(9\ 10\ 11\ 12\ 4)(13\ 14\ 15\ 16\ 4)(17\ 1\ 2\ 3\ 4)$   
 $(6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5) = (6\ 7\ 8\ 9\ 5)(10\ 11\ 12\ 13\ 5)(14\ 15\ 16\ 17\ 5)(1\ 2\ 3\ 4\ 5)$   
 $(7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6) = (7\ 8\ 9\ 10\ 6)(11\ 12\ 13\ 14\ 6)(15\ 16\ 17\ 1\ 6)(2\ 3\ 4\ 5\ 6)$   
 $(8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7) = (8\ 9\ 10\ 11\ 7)(12\ 13\ 14\ 15\ 7)(16\ 17\ 1\ 2\ 7)(3\ 4\ 5\ 6\ 7)$   
 $(9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8) = (9\ 10\ 11\ 12\ 8)(13\ 14\ 15\ 16\ 8)(17\ 1\ 2\ 3\ 8)(4\ 5\ 6\ 7\ 8)$   
 $(10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9) = (10\ 11\ 12\ 13\ 9)(14\ 15\ 16\ 17\ 9)(1\ 2\ 3\ 4\ 9)(5\ 6\ 7\ 8\ 9)$   
 $(11\ 12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10) = (11\ 12\ 13\ 14\ 10)(15\ 16\ 17\ 1\ 10)(2\ 3\ 4\ 5\ 10)(6\ 7\ 8\ 9\ 10)$   
 $(12\ 13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11) = (12\ 13\ 14\ 15\ 11)(16\ 17\ 1\ 2\ 11)(3\ 4\ 5\ 6\ 11)(7\ 8\ 9\ 10\ 11)$   
 $(13\ 14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12) = (13\ 14\ 15\ 16\ 12)(17\ 1\ 2\ 3\ 12)(4\ 5\ 6\ 7\ 12)(8\ 9\ 10\ 11\ 12)$   
 $(14\ 15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13) = (14\ 15\ 16\ 17\ 13)(1\ 2\ 3\ 4\ 13)(5\ 6\ 7\ 8\ 13)(9\ 10\ 11\ 12\ 13)$   
 $(15\ 16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14) = (15\ 16\ 17\ 1\ 14)(2\ 3\ 4\ 5\ 14)(6\ 7\ 8\ 9\ 14)(10\ 11\ 12\ 13\ 14)$   
 $(16\ 17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15) = (16\ 17\ 1\ 2\ 15)(3\ 4\ 5\ 6\ 15)(7\ 8\ 9\ 10\ 15)(11\ 12\ 13\ 14\ 15)$   
 $(17\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16) = (17\ 1\ 2\ 3\ 16)(4\ 5\ 6\ 7\ 16)(8\ 9\ 10\ 11\ 16)(12\ 13\ 14\ 15\ 16)$

**(19c) = (3c) (5c) (5c) (5c) (5c) – Left**

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19) =$

(1 2 3) (1 4 5 6 7) (1 8 9 10 11) (1 12 13 14 15) (1 16 17 18 19)

(2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 1) =  
(2 3 4) (2 5 6 7 8) (2 9 10 11 12) (2 13 14 15 16) (2 17 18 19 1)

(3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 1 2) =  
(3 4 5) (3 6 7 8 9) (3 10 11 12 13) (3 14 15 16 17) (3 18 19 1 2)

(4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 1 2 3) =  
(4 5 6) (4 7 8 9 10) (4 11 12 13 14) (4 15 16 17 18) (4 19 1 2 3)

(5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 1 2 3 4) =  
(5 6 7) (5 8 9 10 11) (5 12 13 14 15) (5 16 17 18 19) (5 1 2 3 4)

(6 7 8 9 10 11 12 13 14 15 16 17 18 19 1 2 3 4 5) =  
(6 7 8) (6 9 10 11 12) (6 13 14 15 16) (6 17 18 19 1) (6 2 3 4 5)

(7 8 9 10 11 12 13 14 15 16 17 18 19 1 2 3 4 5 6) =  
(7 8 9) (7 10 11 12 13) (7 14 15 16 17) (7 18 19 1 2) (7 3 4 5 6)

(8 9 10 11 12 13 14 15 16 17 18 19 1 2 3 4 5 6 7) =  
(8 9 10) (8 11 12 13 14) (8 15 16 17 18) (8 19 1 2 3) (8 4 5 6 7)

(9 10 11 12 13 14 15 16 17 18 19 1 2 3 4 5 6 7 8) =  
(9 10 11) (9 12 13 14 15) (9 16 17 18 19) (9 1 2 3 4) (9 5 6 7 8)

(10 11 12 13 14 15 16 17 18 19 1 2 3 4 5 6 7 8 9) =  
(10 11 12) (10 13 14 15 16) (10 17 18 19 1) (10 2 3 4 5) (10 6 7 8 9)

(11 12 13 14 15 16 17 18 19 1 2 3 4 5 6 7 8 9 10) =  
(11 12 13) (11 14 15 16 17) (11 18 19 1 2) (11 3 4 5 6) (11 7 8 9 10)

(12 13 14 15 16 17 18 19 1 2 3 4 5 6 7 8 9 10 11) =  
(12 13 14) (12 15 16 17 18) (12 19 1 2 3) (12 4 5 6 7) (12 8 9 10 11)

(13 14 15 16 17 18 19 1 2 3 4 5 6 7 8 9 10 11 12) =  
(13 14 15) (13 16 17 18 19) (13 1 2 3 4) (13 5 6 7 8) (13 9 10 11 12)

(14 15 16 17 18 19 1 2 3 4 5 6 7 8 9 10 11 12 13) =  
(14 15 16) (14 17 18 19 1) (14 2 3 4 5) (14 6 7 8 9) (14 10 11 12 13)

(15 16 17 18 19 1 2 3 4 5 6 7 8 9 10 11 12 13 14) =  
(15 16 17) (15 18 19 1 2) (15 3 4 5 6) (15 7 8 9 10) (15 11 12 13 14)

(16 17 18 19 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15) =  
(16 17 18) (16 19 1 2 3) (16 4 5 6 7) (16 8 9 10 11) (16 12 13 14 15)

(17 18 19 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16) =  
(17 18 19) (17 1 2 3 4) (17 5 6 7 8) (17 9 10 11 12) (17 13 14 15 16)

(18 19 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17) =  
(18 19 1) (18 2 3 4 5) (18 6 7 8 9) (18 10 11 12 13) (18 14 15 16 17)

(19 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18) =  
(19 1 2) (19 3 4 5 6) (19 7 8 9 10) (19 11 12 13 14) (19 15 16 17 18)

**(19c) = (5c) (5c) (5c) (3c) – Right**

(1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19) =  
(1 2 3 4 19) (5 6 7 8 19) (9 10 11 12 19) (13 14 15 16 19) (17 18 19)

(2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 1) =

(2 3 4 5 1) (6 7 8 9 1) (10 11 12 13 1) (14 15 16 17 1) (18 19 1)

(3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 1 2) =  
(3 4 5 6 2) (7 8 9 10 2) (11 12 13 14 2) (15 16 17 18 2) (19 1 2)

(4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 1 2 3) =  
(4 5 6 7 3) (8 9 10 11 3) (12 13 14 15 3) (16 17 18 19 3) (1 2 3)

(5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 1 2 3 4) =  
(5 6 7 8 4) (9 10 11 12 4) (13 14 15 16 4) (17 18 19 1 4) (2 3 4)

(6 7 8 9 10 11 12 13 14 15 16 17 18 19 1 2 3 4 5) =  
(6 7 8 9 5) (10 11 12 13 5) (14 15 16 17 5) (18 19 1 2 5) (3 4 5)

(7 8 9 10 11 12 13 14 15 16 17 18 19 1 2 3 4 5 6) =  
(7 8 9 10 6) (11 12 13 14 6) (15 16 17 18 6) (19 1 2 3 6) (4 5 6)

(8 9 10 11 12 13 14 15 16 17 18 19 1 2 3 4 5 6 7) =  
(8 9 10 11 7) (12 13 14 15 7) (16 17 18 19 7) (1 2 3 4 7) (5 6 7)

(9 10 11 12 13 14 15 16 17 18 19 1 2 3 4 5 6 7 8) =  
(9 10 11 12 8) (13 14 15 16 8) (17 18 19 1 8) (2 3 4 5 8) (6 7 8)

(10 11 12 13 14 15 16 17 18 19 1 2 3 4 5 6 7 8 9) =  
(10 11 12 13 9) (14 15 16 17 9) (18 19 1 2 9) (3 4 5 6 9) (7 8 9)

(11 12 13 14 15 16 17 18 19 1 2 3 4 5 6 7 8 9 10) =  
(11 12 13 14 10) (15 16 17 18 10) (19 1 2 3 10) (4 5 6 7 10) (8 9 10)

(12 13 14 15 16 17 18 19 1 2 3 4 5 6 7 8 9 10 11) =  
(12 13 14 15 11) (16 17 18 19 11) (1 2 3 4 11) (5 6 7 8 11) (9 10 11)

(13 14 15 16 17 18 19 1 2 3 4 5 6 7 8 9 10 11 12) =  
(13 14 15 16 12) (17 18 19 1 12) (2 3 4 5 12) (6 7 8 9 12) (10 11 12)

(14 15 16 17 18 19 1 2 3 4 5 6 7 8 9 10 11 12 13) =  
(14 15 16 17 13) (18 19 1 2 13) (3 4 5 6 13) (7 8 9 10 13) (11 12 13)

(15 16 17 18 19 1 2 3 4 5 6 7 8 9 10 11 12 13 14) =  
(15 16 17 18 14) (19 1 2 3 14) (4 5 6 7 14) (8 9 10 11 14) (12 13 14)

(16 17 18 19 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15) =  
(16 17 18 19 15) (1 2 3 4 15) (5 6 7 8 15) (9 10 11 12 15) (13 14 15)

(17 18 19 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16) =  
(17 18 19 1 16) (2 3 4 5 16) (6 7 8 9 16) (10 11 12 13 16) (14 15 16)

(18 19 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17) =  
(18 19 1 2 17) (3 4 5 6 17) (7 8 9 10 17) (11 12 13 14 17) (15 16 17)

(19 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18) =  
(19 1 2 3 18) (4 5 6 7 18) (8 9 10 11 18) (12 13 14 15 18) (16 17 18)

**(21c) = (5c) (5c) (5c) (5c) – Left**

(1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21) =  
(1 2 3 4 5) (1 6 7 8 9) (1 10 11 12 13) (1 14 15 16 17) (1 18 19 20 21)

(2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 1) =  
(2 3 4 5 6) (2 7 8 9 10) (2 11 12 13 14) (2 15 16 17 18) (2 19 20 21 1)

(3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 1 2) =

(3 4 5 6 7) (3 8 9 10 11) (3 12 13 14 15) (3 16 17 18 19) (3 20 21 1 2)

(4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 1 2 3) =  
(4 5 6 7 8) (4 9 10 11 12) (4 13 14 15 16) (4 17 18 19 20) (4 21 1 2 3)

(5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 1 2 3 4) =  
(5 6 7 8 9) (5 10 11 12 13) (5 14 15 16 17) (5 18 19 20 21) (5 1 2 3 4)

(6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 1 2 3 4 5) =  
(6 7 8 9 10) (6 11 12 13 14) (6 15 16 17 18) (6 19 20 21 1) (6 2 3 4 5)

(7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 1 2 3 4 5 6) =  
(7 8 9 10 11) (7 12 13 14 15) (7 16 17 18 19) (7 20 21 1 2) (7 3 4 5 6)

(8 9 10 11 12 13 14 15 16 17 18 19 20 21 1 2 3 4 5 6 7) =  
(8 9 10 11 12) (8 13 14 15 16) (8 17 18 19 20) (8 21 1 2 3) (8 4 5 6 7)

(9 10 11 12 13 14 15 16 17 18 19 20 21 1 2 3 4 5 6 7 8) =  
(9 10 11 12 13) (9 14 15 16 17) (9 18 19 20 21) (9 1 2 3 4) (9 5 6 7 8)

(10 11 12 13 14 15 16 17 18 19 20 21 1 2 3 4 5 6 7 8 9) =  
(10 11 12 13 14) (10 15 16 17 18) (10 19 20 21 1) (10 2 3 4 5) (10 6 7 8 9)

(11 12 13 14 15 16 17 18 19 20 21 1 2 3 4 5 6 7 8 9 10) =  
(11 12 13 14 15) (11 16 17 18 19) (11 20 21 1 2) (11 3 4 5 6) (11 7 8 9 10)

(12 13 14 15 16 17 18 19 20 21 1 2 3 4 5 6 7 8 9 10 11) =  
(12 13 14 15 16) (12 17 18 19 20) (12 21 1 2 3) (12 4 5 6 7) (12 8 9 10 11)

(13 14 15 16 17 18 19 20 21 1 2 3 4 5 6 7 8 9 10 11 12) =  
(13 14 15 16 17) (13 18 19 20 21) (13 1 2 3 4) (13 5 6 7 8) (13 9 10 11 12)

(14 15 16 17 18 19 20 21 1 2 3 4 5 6 7 8 9 10 11 12 13) =  
(14 15 16 17 18) (14 19 20 21 1) (14 2 3 4 5) (14 6 7 8 9) (14 10 11 12 13)

(15 16 17 18 19 20 21 1 2 3 4 5 6 7 8 9 10 11 12 13 14) =  
(15 16 17 18 19) (15 20 21 1 2) (15 3 4 5 6) (15 7 8 9 10) (15 11 12 13 14)

(16 17 18 19 20 21 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15) =  
(16 17 18 19 20) (16 21 1 2 3) (16 4 5 6 7) (16 8 9 10 11) (16 12 13 14 15)

(17 18 19 20 21 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16) =  
(17 18 19 20 21) (17 1 2 3 4) (17 5 6 7 8) (17 9 10 11 12) (17 13 14 15 16)

(18 19 20 21 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17) =  
(18 19 20 21 1) (18 2 3 4 5) (18 6 7 8 9) (18 10 11 12 13) (18 14 15 16 17)

(19 20 21 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18) =  
(19 20 21 1 2) (19 3 4 5 6) (19 7 8 9 10) (19 11 12 13 14) (19 15 16 17 18)

(20 21 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19) =  
(20 21 1 2 3) (20 4 5 6 7) (20 8 9 10 11) (20 12 13 14 15) (20 16 17 18 19)

(21 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20) =  
(21 1 2 3 4) (21 5 6 7 8) (21 9 10 11 12) (21 13 14 15 16) (21 17 18 19 20)

### **(21c) = (5c) (5c) (5c) (5c) – Right**

(1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21) =  
(1 2 3 4 21) (5 6 7 8 21) (9 10 11 12 21) (13 14 15 16 21) (17 18 19 20 21)

(2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 1) =  
(2 3 4 5 1) (6 7 8 9 1) (10 11 12 13 1) (14 15 16 17 1) (18 19 20 21 1)

$(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2) =$   
 $(3\ 4\ 5\ 6\ 2)(7\ 8\ 9\ 10\ 2)(11\ 12\ 13\ 14\ 2)(15\ 16\ 17\ 18\ 2)(19\ 20\ 21\ 1\ 2)$

$(4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3) =$   
 $(4\ 5\ 6\ 7\ 3)(8\ 9\ 10\ 11\ 3)(12\ 13\ 14\ 15\ 3)(16\ 17\ 18\ 19\ 3)(20\ 21\ 1\ 2\ 3)$

$(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4) =$   
 $(5\ 6\ 7\ 8\ 4)(9\ 10\ 11\ 12\ 4)(13\ 14\ 15\ 16\ 4)(17\ 18\ 19\ 20\ 4)(21\ 1\ 2\ 3\ 4)$

$(6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5) =$   
 $(6\ 7\ 8\ 9\ 5)(10\ 11\ 12\ 13\ 5)(14\ 15\ 16\ 17\ 5)(18\ 19\ 20\ 21\ 5)(1\ 2\ 3\ 4\ 5)$

$(7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6) =$   
 $(7\ 8\ 9\ 10\ 6)(11\ 12\ 13\ 14\ 6)(15\ 16\ 17\ 18\ 6)(19\ 20\ 21\ 1\ 6)(2\ 3\ 4\ 5\ 6)$

$(8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7) =$   
 $(8\ 9\ 10\ 11\ 7)(12\ 13\ 14\ 15\ 7)(16\ 17\ 18\ 19\ 7)(20\ 21\ 1\ 2\ 7)(3\ 4\ 5\ 6\ 7)$

$(9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8) =$   
 $(9\ 10\ 11\ 12\ 8)(13\ 14\ 15\ 16\ 8)(17\ 18\ 19\ 20\ 8)(21\ 1\ 2\ 3\ 8)(4\ 5\ 6\ 7\ 8)$

$(10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9) =$   
 $(10\ 11\ 12\ 13\ 9)(14\ 15\ 16\ 17\ 9)(18\ 19\ 20\ 21\ 9)(1\ 2\ 3\ 4\ 9)(5\ 6\ 7\ 8\ 9)$

$(11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10) =$   
 $(11\ 12\ 13\ 14\ 10)(15\ 16\ 17\ 18\ 10)(19\ 20\ 21\ 1\ 10)(2\ 3\ 4\ 5\ 10)(6\ 7\ 8\ 9\ 10)$

$(12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11) =$   
 $(12\ 13\ 14\ 15\ 11)(16\ 17\ 18\ 19\ 11)(20\ 21\ 1\ 2\ 11)(3\ 4\ 5\ 6\ 11)(7\ 8\ 9\ 10\ 11)$

$(13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12) =$   
 $(13\ 14\ 15\ 16\ 12)(17\ 18\ 19\ 20\ 12)(21\ 1\ 2\ 3\ 12)(4\ 5\ 6\ 7\ 12)(8\ 9\ 10\ 11\ 12)$

$(14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13) =$   
 $(14\ 15\ 16\ 17\ 13)(18\ 19\ 20\ 21\ 13)(1\ 2\ 3\ 4\ 13)(5\ 6\ 7\ 8\ 13)(9\ 10\ 11\ 12\ 13)$

$(15\ 16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14) =$   
 $(15\ 16\ 17\ 18\ 14)(19\ 20\ 21\ 1\ 14)(2\ 3\ 4\ 5\ 14)(6\ 7\ 8\ 9\ 14)(10\ 11\ 12\ 13\ 14)$

$(16\ 17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15) =$   
 $(16\ 17\ 18\ 19\ 15)(20\ 21\ 1\ 2\ 15)(3\ 4\ 5\ 6\ 15)(7\ 8\ 9\ 10\ 15)(11\ 12\ 13\ 14\ 15)$

$(17\ 18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16) =$   
 $(17\ 18\ 19\ 20\ 16)(21\ 1\ 2\ 3\ 16)(4\ 5\ 6\ 7\ 16)(8\ 9\ 10\ 11\ 16)(12\ 13\ 14\ 15\ 16)$

$(18\ 19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17) =$   
 $(18\ 19\ 20\ 21\ 17)(1\ 2\ 3\ 4\ 17)(5\ 6\ 7\ 8\ 17)(9\ 10\ 11\ 12\ 17)(13\ 14\ 15\ 16\ 17)$

$(19\ 20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18) =$   
 $(19\ 20\ 21\ 1\ 18)(2\ 3\ 4\ 5\ 18)(6\ 7\ 8\ 9\ 18)(10\ 11\ 12\ 13\ 18)(14\ 15\ 16\ 17\ 18)$

$(20\ 21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19) =$   
 $(20\ 21\ 1\ 2\ 19)(3\ 4\ 5\ 6\ 19)(7\ 8\ 9\ 10\ 19)(11\ 12\ 13\ 14\ 19)(15\ 16\ 17\ 18\ 19)$

$(21\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20) =$   
 $(21\ 1\ 2\ 3\ 20)(4\ 5\ 6\ 7\ 20)(8\ 9\ 10\ 11\ 20)(12\ 13\ 14\ 15\ 20)(16\ 17\ 18\ 19\ 20)$

### **(23c) = (3c) (5c) (5c) (5c) (5c) – Left**

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23) =$   
 $(1\ 2\ 3)(1\ 4\ 5\ 6\ 7)(1\ 8\ 9\ 10\ 11)(1\ 12\ 13\ 14\ 15)(1\ 16\ 17\ 18\ 19)(1\ 20\ 21\ 22\ 23)$

$(\underline{2} \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1) =$   
 $(\underline{2} \ 3 \ 4) (\underline{2} \ 5 \ 6 \ 7 \ 8) (\underline{2} \ 9 \ 10 \ 11 \ 12) (\underline{2} \ 13 \ 14 \ 15 \ 16) (\underline{2} \ 17 \ 18 \ 19 \ 20) (\underline{2} \ 21 \ 22 \ 23 \ 1)$

$(\underline{3} \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2) =$   
 $(\underline{3} \ 4 \ 5) (\underline{3} \ 6 \ 7 \ 8 \ 9) (\underline{3} \ 10 \ 11 \ 12 \ 13) (\underline{3} \ 14 \ 15 \ 16 \ 17) (\underline{3} \ 18 \ 19 \ 20 \ 21) (\underline{3} \ 22 \ 23 \ 1 \ 2)$

$(\underline{4} \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3) =$   
 $(\underline{4} \ 5 \ 6) (\underline{4} \ 7 \ 8 \ 9 \ 10) (\underline{4} \ 11 \ 12 \ 13 \ 14) (\underline{4} \ 15 \ 16 \ 17 \ 18) (\underline{4} \ 19 \ 20 \ 21 \ 22) (\underline{4} \ 23 \ 1 \ 2 \ 3)$

$(\underline{5} \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4) =$   
 $(\underline{5} \ 6 \ 7) (\underline{5} \ 8 \ 9 \ 10 \ 11) (\underline{5} \ 12 \ 13 \ 14 \ 15) (\underline{5} \ 16 \ 17 \ 18 \ 19) (\underline{5} \ 20 \ 21 \ 22 \ 23) (\underline{5} \ 1 \ 2 \ 3 \ 4)$

$(\underline{6} \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5) =$   
 $(\underline{6} \ 7 \ 8) (\underline{6} \ 9 \ 10 \ 11 \ 12) (\underline{6} \ 13 \ 14 \ 15 \ 16) (\underline{6} \ 17 \ 18 \ 19 \ 20) (\underline{6} \ 21 \ 22 \ 23 \ 1) (\underline{6} \ 2 \ 3 \ 4 \ 5)$

$(\underline{7} \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6) =$   
 $(\underline{7} \ 8 \ 9) (\underline{7} \ 10 \ 11 \ 12 \ 13) (\underline{7} \ 14 \ 15 \ 16 \ 17) (\underline{7} \ 18 \ 19 \ 20 \ 21) (\underline{7} \ 22 \ 23 \ 1 \ 2) (\underline{7} \ 3 \ 4 \ 5 \ 6)$

$(\underline{8} \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) =$   
 $(\underline{8} \ 9 \ 10) (\underline{8} \ 11 \ 12 \ 13 \ 14) (\underline{8} \ 15 \ 16 \ 17 \ 18) (\underline{8} \ 19 \ 20 \ 21 \ 22) (\underline{8} \ 23 \ 1 \ 2 \ 3) (\underline{8} \ 4 \ 5 \ 6 \ 7)$

$(\underline{9} \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) =$   
 $(\underline{9} \ 10 \ 11) (\underline{9} \ 12 \ 13 \ 14 \ 15) (\underline{9} \ 16 \ 17 \ 18 \ 19) (\underline{9} \ 20 \ 21 \ 22 \ 23) (\underline{9} \ 1 \ 2 \ 3 \ 4) (\underline{9} \ 5 \ 6 \ 7 \ 8)$

$(\underline{10} \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9) =$   
 $(\underline{10} \ 11 \ 12) (\underline{10} \ 13 \ 14 \ 15 \ 16) (\underline{10} \ 17 \ 18 \ 19 \ 20) (\underline{10} \ 21 \ 22 \ 23 \ 1) (\underline{10} \ 2 \ 3 \ 4 \ 5) (\underline{10} \ 6 \ 7 \ 8 \ 9)$

$(\underline{11} \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10) =$   
 $(\underline{11} \ 12 \ 13) (\underline{11} \ 14 \ 15 \ 16 \ 17) (\underline{11} \ 18 \ 19 \ 20 \ 21) (\underline{11} \ 22 \ 23 \ 1 \ 2) (\underline{11} \ 3 \ 4 \ 5 \ 6) (\underline{11} \ 7 \ 8 \ 9 \ 10)$

$(\underline{12} \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11) =$   
 $(\underline{12} \ 13 \ 14) (\underline{12} \ 15 \ 16 \ 17 \ 18) (\underline{12} \ 19 \ 20 \ 21 \ 22) (\underline{12} \ 23 \ 1 \ 2 \ 3) (\underline{12} \ 4 \ 5 \ 6 \ 7) (\underline{12} \ 8 \ 9 \ 10 \ 11)$

$(\underline{13} \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12) =$   
 $(\underline{13} \ 14 \ 15) (\underline{13} \ 16 \ 17 \ 18 \ 19) (\underline{13} \ 20 \ 21 \ 22 \ 23) (\underline{13} \ 1 \ 2 \ 3 \ 4) (\underline{13} \ 5 \ 6 \ 7 \ 8) (\underline{13} \ 9 \ 10 \ 11 \ 12)$

$(\underline{14} \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13) =$   
 $(\underline{14} \ 15 \ 16) (\underline{14} \ 17 \ 18 \ 19 \ 20) (\underline{14} \ 21 \ 22 \ 23 \ 1) (\underline{14} \ 2 \ 3 \ 4 \ 5) (\underline{14} \ 6 \ 7 \ 8 \ 9) (\underline{14} \ 10 \ 11 \ 12 \ 13)$

$(\underline{15} \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14) =$   
 $(\underline{15} \ 16 \ 17) (\underline{15} \ 18 \ 19 \ 20 \ 21) (\underline{15} \ 22 \ 23 \ 1 \ 2) (\underline{15} \ 3 \ 4 \ 5 \ 6) (\underline{15} \ 7 \ 8 \ 9 \ 10) (\underline{15} \ 11 \ 12 \ 13 \ 14)$

$(\underline{16} \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15) =$   
 $(\underline{16} \ 17 \ 18) (\underline{16} \ 19 \ 20 \ 21 \ 22) (\underline{16} \ 23 \ 1 \ 2 \ 3) (\underline{16} \ 4 \ 5 \ 6 \ 7) (\underline{16} \ 8 \ 9 \ 10 \ 11) (\underline{16} \ 12 \ 13 \ 14 \ 15)$

$(\underline{17} \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16) =$   
 $(\underline{17} \ 18 \ 19) (\underline{17} \ 20 \ 21 \ 22 \ 23) (\underline{17} \ 1 \ 2 \ 3 \ 4) (\underline{17} \ 5 \ 6 \ 7 \ 8) (\underline{17} \ 9 \ 10 \ 11 \ 12) (\underline{17} \ 13 \ 14 \ 15 \ 16)$

$(\underline{18} \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17) =$   
 $(\underline{18} \ 19 \ 20) (\underline{18} \ 21 \ 22 \ 23 \ 1) (\underline{18} \ 2 \ 3 \ 4 \ 5) (\underline{18} \ 6 \ 7 \ 8 \ 9) (\underline{18} \ 10 \ 11 \ 12 \ 13) (\underline{18} \ 14 \ 15 \ 16 \ 17)$

$(\underline{19} \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18) =$   
 $(\underline{19} \ 20 \ 21) (\underline{19} \ 22 \ 23 \ 1 \ 2) (\underline{19} \ 3 \ 4 \ 5 \ 6) (\underline{19} \ 7 \ 8 \ 9 \ 10) (\underline{19} \ 11 \ 12 \ 13 \ 14) (\underline{19} \ 15 \ 16 \ 17 \ 18)$

$(\underline{20} \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19) =$   
 $(\underline{20} \ 21 \ 22) (\underline{20} \ 23 \ 1 \ 2 \ 3) (\underline{20} \ 4 \ 5 \ 6 \ 7) (\underline{20} \ 8 \ 9 \ 10 \ 11) (\underline{20} \ 12 \ 13 \ 14 \ 15) (\underline{20} \ 16 \ 17 \ 18 \ 19)$

$(\underline{21} \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20) =$   
 $(\underline{21} \ 22 \ 23) (\underline{21} \ 1 \ 2 \ 3 \ 4) (\underline{21} \ 5 \ 6 \ 7 \ 8) (\underline{21} \ 9 \ 10 \ 11 \ 12) (\underline{21} \ 13 \ 14 \ 15 \ 16) (\underline{21} \ 17 \ 18 \ 19 \ 20)$

$(\underline{22} \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21) =$   
 $(\underline{22} \ 23 \ 1) (\underline{22} \ 2 \ 3 \ 4 \ 5) (\underline{22} \ 6 \ 7 \ 8 \ 9) (\underline{22} \ 10 \ 11 \ 12 \ 13) (\underline{22} \ 14 \ 15 \ 16 \ 17) (\underline{22} \ 18 \ 19 \ 20 \ 21)$

$$\begin{aligned} & (\underline{23} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22) = \\ & (\underline{23} \ 1 \ 2) (\underline{23} \ 3 \ 4 \ 5 \ 6) (\underline{23} \ 7 \ 8 \ 9 \ 10) (\underline{23} \ 11 \ 12 \ 13 \ 14) (\underline{23} \ 15 \ 16 \ 17 \ 18) (\underline{23} \ 19 \ 20 \ 21 \ 22) \end{aligned}$$

**(23c) = (5c) (5c) (5c) (5c) (5c) (3c) – Right**

$$(1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23) = \\ (1 \ 2 \ 3 \ 4 \ 23) (5 \ 6 \ 7 \ 8 \ 23) (9 \ 10 \ 11 \ 12 \ 23) (13 \ 14 \ 15 \ 16 \ 23) (17 \ 18 \ 19 \ 20 \ 23) (21 \ 22 \ 23)$$

$$(2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1) = \\ (2\ 3\ 4\ 5\ 1)(6\ 7\ 8\ 9\ 1)(10\ 11\ 12\ 13\ 1)(14\ 15\ 16\ 17\ 1)(18\ 19\ 20\ 21\ 1)(22\ 23\ 1)$$

$$(3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2) = \\ (3\ 4\ 5\ 6\ 2)(7\ 8\ 9\ 10\ 2)(11\ 12\ 13\ 14\ 2)(15\ 16\ 17\ 18\ 2)(19\ 20\ 21\ 22\ 2)(23\ 1\ 2)$$

$$(4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2\ \underline{3}) = \\ (4\ 5\ 6\ 7\ \underline{3})(8\ 9\ 10\ 11\ \underline{3})(12\ 13\ 14\ 15\ \underline{3})(16\ 17\ 18\ 19\ \underline{3})(20\ 21\ 22\ 23\ \underline{3})(1\ 2\ 3)$$

$$(5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2\ 3\ 4) = \\ (5\ 6\ 7\ 8\ 4)(9\ 10\ 11\ 12\ 4)(13\ 14\ 15\ 16\ 4)(17\ 18\ 19\ 20\ 4)(21\ 22\ 23\ 1\ 4)(2\ 3\ 4)$$

$$(6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2\ 3\ 4\ \underline{5}) = \\ (6\ 7\ 8\ 9\ \underline{5})(10\ 11\ 12\ 13\ \underline{5})(14\ 15\ 16\ 17\ \underline{5})(18\ 19\ 20\ 21\ \underline{5})(22\ 23\ 1\ 2\ \underline{5})(3\ 4\ \underline{5})$$

$$(7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2\ 3\ 4\ 5\ \underline{6}) = \\ (7\ 8\ 9\ 10\ \underline{6})(11\ 12\ 13\ 14\ \underline{6})(15\ 16\ 17\ 18\ \underline{6})(19\ 20\ 21\ 22\ \underline{6})(23\ 1\ 2\ 3\ \underline{6})(4\ 5\ \underline{6})$$

$$(8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2\ 3\ 4\ 5\ 6\ 7) = \\ (8\ 9\ 10\ 11\ 7)\ (12\ 13\ 14\ 15\ 7)\ (16\ 17\ 18\ 19\ 7)\ (20\ 21\ 22\ 23\ 7)\ (1\ 2\ 3\ 4\ 7)\ (5\ 6\ 7)$$

$$(9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ \underline{8}) = \\ (9\ 10\ 11\ 12\ \underline{8}) (13\ 14\ 15\ 16\ \underline{8}) (17\ 18\ 19\ 20\ \underline{8}) (21\ 22\ 23\ 1\ \underline{8}) (2\ 3\ 4\ 5\ \underline{8}) (6\ 7\ \underline{8})$$

$$(10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9) = \\ (10\ 11\ 12\ 13\ 9)\ (14\ 15\ 16\ 17\ 9)\ (18\ 19\ 20\ 21\ 9)\ (22\ 23\ 1\ 2\ 9)\ (3\ 4\ 5\ 6\ 9)\ (7\ 8\ 9)$$

$$(11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ \underline{10}) = \\ (11\ 12\ 13\ 14\ \underline{10}) (15\ 16\ 17\ 18\ \underline{10}) (19\ 20\ 21\ 22\ \underline{10}) (23\ 1\ 2\ 3\ \underline{10}) (4\ 5\ 6\ 7\ \underline{10}) (8\ 9\ \underline{10})$$

$$(12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22\ 23\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ \underline{11}) = \\ (12\ 13\ 14\ 15\ \underline{11}) (16\ 17\ 18\ 19\ \underline{11}) (20\ 21\ 22\ 23\ \underline{11}) (1\ 2\ 3\ 4\ \underline{11}) (5\ 6\ 7\ 8\ \underline{11}) (9\ 10\ \underline{11})$$

$$(13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12) = \\ (13 \ 14 \ 15 \ 16 \ 12) (17 \ 18 \ 19 \ 20 \ 12) (21 \ 22 \ 23 \ 1 \ 12) (2 \ 3 \ 4 \ 5 \ 12) (6 \ 7 \ 8 \ 9 \ 12) (10 \ 11 \ 12)$$

$$(14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13) = \\ (14 \ 15 \ 16 \ 17 \ 13) (18 \ 19 \ 20 \ 21 \ 13) (22 \ 23 \ 1 \ 2 \ 13) (3 \ 4 \ 5 \ 6 \ 13) (7 \ 8 \ 9 \ 10 \ 13) (11 \ 12 \ 13)$$

$$(15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14) = \\ (15 \ 16 \ 17 \ 18 \ 14) (19 \ 20 \ 21 \ 22 \ 14) (23 \ 1 \ 2 \ 3 \ 14) (4 \ 5 \ 6 \ 7 \ 14) (8 \ 9 \ 10 \ 11 \ 14) (12 \ 13 \ 14)$$

$$(16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15) = \\ (16 \ 17 \ 18 \ 19 \ 15) (20 \ 21 \ 22 \ 23 \ 15) (1 \ 2 \ 3 \ 4 \ 15) (5 \ 6 \ 7 \ 8 \ 15) (9 \ 10 \ 11 \ 12 \ 15) (13 \ 14 \ 15)$$

$$(17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16) = \\ (17 \ 18 \ 19 \ 20 \ 16) (21 \ 22 \ 23 \ 1 \ 16) (2 \ 3 \ 4 \ 5 \ 16) (6 \ 7 \ 8 \ 9 \ 16) (10 \ 11 \ 12 \ 13 \ 16) (14 \ 15 \ 16)$$

$$(18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17) = \\ (18 \ 19 \ 20 \ 21 \ 17) (22 \ 23 \ 1 \ 2 \ 17) (3 \ 4 \ 5 \ 6 \ 17) (7 \ 8 \ 9 \ 10 \ 17) (11 \ 12 \ 13 \ 14 \ 17) (15 \ 16 \ 17)$$

$$(19 \ 20 \ 21 \ 22 \ 23 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18) = \\ (19 \ 20 \ 21 \ 22 \ 18) (23 \ 1 \ 2 \ 3 \ 18) (4 \ 5 \ 6 \ 7 \ 18) (8 \ 9 \ 10 \ 11 \ 18) (12 \ 13 \ 14 \ 15 \ 18) (16 \ 17 \ 18)$$

$(20\ 21\ 22\ 23\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19) =$   
 $(20\ 21\ 22\ 23\ 19)\ (1\ 2\ 3\ 4\ 19)\ (5\ 6\ 7\ 8\ 19)\ (9\ 10\ 11\ 12\ 19)\ (13\ 14\ 15\ 16\ 19)\ (17\ 18\ 19)$

$(21\ 22\ 23\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20) =$   
 $(21\ 22\ 23\ 1\ 20)\ (2\ 3\ 4\ 5\ 20)\ (6\ 7\ 8\ 9\ 20)\ (10\ 11\ 12\ 13\ 20)\ (14\ 15\ 16\ 17\ 20)\ (18\ 19\ 20)$

$(22\ 23\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21) =$   
 $(22\ 23\ 1\ 2\ 21)\ (3\ 4\ 5\ 6\ 21)\ (7\ 8\ 9\ 10\ 21)\ (11\ 12\ 13\ 14\ 21)\ (15\ 16\ 17\ 18\ 21)\ (19\ 20\ 21)$

$(23\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 21\ 22) =$   
 $(23\ 1\ 2\ 3\ 22)\ (4\ 5\ 6\ 7\ 22)\ (8\ 9\ 10\ 11\ 22)\ (12\ 13\ 14\ 15\ 22)\ (16\ 17\ 18\ 19\ 22)\ (20\ 21\ 22)$

# Even-Cycle Decomposition

## (4c) = (2c) (3c) – Left

$$\begin{aligned}(\underline{1} \ 2 \ 3 \ 4) &= (\underline{1} \ 2) (\underline{1} \ 3 \ 4) \\(\underline{2} \ 3 \ 4 \ 1) &= (\underline{2} \ 3) (\underline{2} \ 4 \ 1) \\(\underline{3} \ 4 \ 1 \ 2) &= (\underline{3} \ 4) (\underline{3} \ 1 \ 2) \\(\underline{4} \ 1 \ 2 \ 3) &= (\underline{4} \ 1) (\underline{4} \ 2 \ 3)\end{aligned}$$

## (4c) = (3c) (2c) – Right

$$\begin{aligned}(\underline{1} \ 2 \ 3 \ \underline{4}) &= (\underline{1} \ 2 \ \underline{4}) (\underline{3} \ \underline{4}) \\(\underline{2} \ 3 \ 4 \ \underline{1}) &= (\underline{2} \ 3 \ \underline{1}) (\underline{4} \ \underline{1}) \\(\underline{3} \ 4 \ 1 \ \underline{2}) &= (\underline{3} \ 4 \ \underline{2}) (\underline{1} \ \underline{2}) \\(\underline{4} \ 1 \ 2 \ \underline{3}) &= (\underline{4} \ 1 \ \underline{3}) (\underline{2} \ \underline{3})\end{aligned}$$

## (6c) = (2c) (5c) – Left

$$\begin{aligned}(\underline{1} \ 2 \ 3 \ 4 \ 5 \ 6) &= (\underline{1} \ 2) (\underline{1} \ 3 \ 4 \ 5 \ 6) \\(\underline{2} \ 3 \ 4 \ 5 \ 6 \ 1) &= (\underline{2} \ 3) (\underline{2} \ 4 \ 5 \ 6 \ 1) \\(\underline{3} \ 4 \ 5 \ 6 \ 1 \ 2) &= (\underline{3} \ 4) (\underline{3} \ 5 \ 6 \ 1 \ 2) \\(\underline{4} \ 5 \ 6 \ 1 \ 2 \ 3) &= (\underline{4} \ 5) (\underline{4} \ 6 \ 1 \ 2 \ 3) \\(\underline{5} \ 6 \ 1 \ 2 \ 3 \ 4) &= (\underline{5} \ 6) (\underline{5} \ 1 \ 2 \ 3 \ 4) \\(\underline{6} \ 1 \ 2 \ 3 \ 4 \ 5) &= (\underline{6} \ 1) (\underline{6} \ 2 \ 3 \ 4 \ 5)\end{aligned}$$

## (6c) = (5c) (2c) – Right

$$\begin{aligned}(\underline{1} \ 2 \ 3 \ 4 \ 5 \ 6) &= (\underline{1} \ 2 \ 3 \ 4 \ \underline{6}) (\underline{5} \ \underline{6}) \\(\underline{2} \ 3 \ 4 \ 5 \ 6 \ \underline{1}) &= (\underline{2} \ 3 \ 4 \ 5 \ \underline{1}) (\underline{6} \ \underline{1}) \\(\underline{3} \ 4 \ 5 \ 6 \ 1 \ \underline{2}) &= (\underline{3} \ 4) (\underline{3} \ 5 \ 6 \ 1 \ \underline{2}) \\(\underline{4} \ 5 \ 6 \ 1 \ 2 \ \underline{3}) &= (\underline{4} \ 5) (\underline{4} \ 6 \ 1 \ 2 \ \underline{3}) \\(\underline{5} \ 6 \ 1 \ 2 \ 3 \ \underline{4}) &= (\underline{5} \ 6) (\underline{5} \ 1 \ 2 \ 3 \ \underline{4}) \\(\underline{6} \ 1 \ 2 \ 3 \ 4 \ \underline{5}) &= (\underline{6} \ 1 \ 2 \ 3 \ \underline{5}) (\underline{4} \ \underline{5})\end{aligned}$$

## (8c) = (2c) (7c) – Left

$$\begin{aligned}(\underline{1} \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) &= (\underline{1} \ 2) (\underline{1} \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) \\(\underline{2} \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 1) &= (\underline{2} \ 3) (\underline{2} \ 4 \ 5 \ 6 \ 7 \ 8 \ 1) \\(\underline{3} \ 4 \ 5 \ 6 \ 7 \ 8 \ 1 \ 2) &= (\underline{3} \ 4) (\underline{3} \ 5 \ 6 \ 7 \ 8 \ 1 \ 2) \\(\underline{4} \ 5 \ 6 \ 7 \ 8 \ 1 \ 2 \ 3) &= (\underline{4} \ 5) (\underline{4} \ 6 \ 7 \ 8 \ 1 \ 2 \ 3) \\(\underline{5} \ 6 \ 7 \ 8 \ 1 \ 2 \ 3 \ 4) &= (\underline{5} \ 6) (\underline{5} \ 7 \ 8 \ 1 \ 2 \ 3 \ 4) \\(\underline{6} \ 7 \ 8 \ 1 \ 2 \ 3 \ 4 \ 5) &= (\underline{6} \ 7) (\underline{6} \ 8 \ 1 \ 2 \ 3 \ 4 \ 5) \\(\underline{7} \ 8 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6) &= (\underline{7} \ 8) (\underline{7} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6) \\(\underline{8} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) &= (\underline{8} \ 1) (\underline{8} \ 2 \ 3 \ 4 \ 5 \ 6 \ 7)\end{aligned}$$

## (8c) = (7c) (2c) – Right

$$\begin{aligned}(\underline{1} \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ \underline{8}) &= (\underline{1} \ 2 \ 3 \ 4 \ 5 \ 6 \ 8) (\underline{7} \ \underline{8}) \\(\underline{2} \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ \underline{1}) &= (\underline{2} \ 3 \ 4 \ 5 \ 6 \ 7 \ \underline{1}) (\underline{8} \ \underline{1}) \\(\underline{3} \ 4 \ 5 \ 6 \ 7 \ 8 \ 1 \ \underline{2}) &= (\underline{3} \ 4 \ 5 \ 6 \ 7 \ 8 \ \underline{2}) (\underline{1} \ \underline{2}) \\(\underline{4} \ 5 \ 6 \ 7 \ 8 \ 1 \ 2 \ \underline{3}) &= (\underline{4} \ 5 \ 6 \ 7 \ 8 \ 1 \ \underline{3}) (\underline{2} \ \underline{3}) \\(\underline{5} \ 6 \ 7 \ 8 \ 1 \ 2 \ 3 \ \underline{4}) &= (\underline{5} \ 6 \ 7 \ 8 \ 1 \ 2 \ \underline{4}) (\underline{3} \ \underline{4}) \\(\underline{6} \ 7 \ 8 \ 1 \ 2 \ 3 \ 4 \ \underline{5}) &= (\underline{6} \ 7 \ 8 \ 1 \ 2 \ 3 \ \underline{5}) (\underline{4} \ \underline{5}) \\(\underline{7} \ 8 \ 1 \ 2 \ 3 \ 4 \ 5 \ \underline{6}) &= (\underline{7} \ 8 \ 1 \ 2 \ 3 \ 4 \ \underline{6}) (\underline{5} \ \underline{6}) \\(\underline{8} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ \underline{7}) &= (\underline{8} \ 1 \ 2 \ 3 \ 4 \ 5 \ \underline{7}) (\underline{6} \ \underline{7})\end{aligned}$$

## (10c) = (2c) (9c) – Left





$$\begin{aligned} (13 \ 14 \ 15 \ 16 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12) &= (13 \ 14 \ 15 \ 16 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 12) (11 \ 12) \\ (14 \ 15 \ 16 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13) &= (14 \ 15 \ 16 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 13) (12 \ 13) \\ (15 \ 16 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14) &= (15 \ 16 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 14) (13 \ 14) \\ (16 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15) &= (16 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 15) (14 \ 15) \end{aligned}$$

$$(18c) = (2c) (17c) - \text{Left}$$

**(18c) = (17c) (2c) – Right**

$$\begin{aligned}
& (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18) = (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18) (17 \ 18) \\
& (2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1) = (2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 1) (18 \ 1) \\
& (3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2) = (3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 2) (1 \ 2) \\
& (4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3) = (4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 3) (2 \ 3) \\
& (5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4) = (5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 4) (3 \ 4) \\
& (6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5) = (6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 5) (4 \ 5) \\
& (7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6) = (7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 6) (5 \ 6) \\
& (8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) = (8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 7) (6 \ 7) \\
& (9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) = (9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 8) (7 \ 8) \\
& (10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9) = (10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 9) (8 \ 9) \\
& (11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10) = (11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 10) (9 \ 10) \\
& (12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11) = (12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 11) (10 \ 11) \\
& (13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12) = (13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 12) (11 \ 12) \\
& (14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13) = (14 \ 15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 13) (12 \ 13) \\
& (15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14) = (15 \ 16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 14) (13 \ 14) \\
& (16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15) = (16 \ 17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 15) (14 \ 15) \\
& (17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16) = (17 \ 18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 16) (15 \ 16) \\
& (18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17) = (18 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 17) (16 \ 17)
\end{aligned}$$

$$(20c) = (2c) (19c) - \text{Left}$$

$$(1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20) = \\ (1 \ 2) (1 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20)$$

$$\begin{aligned} & (\underline{2} \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1) = \\ & (\underline{2} \ 3) (\underline{2} \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1) \end{aligned}$$

$$\begin{aligned} & (\underline{3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2}) = \\ & (\underline{3 \ 4}) (\underline{3 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2}) \end{aligned}$$

$$\begin{aligned} & (4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3) = \\ & (4 \ 5) (4 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3) \end{aligned}$$

$(\underline{5} \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4) =$   
 $(\underline{5} \ 6) (\underline{5} \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4)$

$(\underline{6} \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5) =$   
 $(\underline{6} \ 7) (\underline{6} \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5)$

$(\underline{7} \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6) =$   
 $(\underline{7} \ 8) (\underline{7} \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6)$

$(\underline{8} \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) =$   
 $(\underline{8} \ 9) (\underline{8} \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7)$

$(\underline{9} \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) =$   
 $(\underline{9} \ 10) (\underline{9} \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8)$

$(\underline{10} \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9) =$   
 $(\underline{10} \ 11) (\underline{10} \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9)$

$(\underline{11} \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10) =$   
 $(\underline{11} \ 12) (\underline{11} \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10)$

$(\underline{12} \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11) =$   
 $(\underline{12} \ 13) (\underline{12} \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11)$

$(\underline{13} \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12) =$   
 $(\underline{13} \ 14) (\underline{13} \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12)$

$(\underline{14} \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13) =$   
 $(\underline{14} \ 15) (\underline{14} \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13)$

$(\underline{15} \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14) =$   
 $(\underline{15} \ 16) (\underline{15} \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14)$

$(\underline{16} \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15) =$   
 $(\underline{16} \ 17) (\underline{16} \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15)$

$(\underline{17} \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16) =$   
 $(\underline{17} \ 18) (\underline{17} \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16)$

$(\underline{18} \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17) =$   
 $(\underline{18} \ 19) (\underline{18} \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17)$

$(\underline{19} \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18) =$   
 $(\underline{19} \ 20) (\underline{19} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18)$

$(\underline{20} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19) =$   
 $(\underline{20} \ 1) (\underline{20} \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19)$

**(20c) = (19c) (2c) – Right**  
 $(1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20) =$   
 $(1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 20) (19 \ 20)$

$(2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1) =$   
 $(2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 1) (20 \ 1)$

$(3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2) =$   
 $(3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 2) (1 \ 2)$

$(4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3) =$   
 $(4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 3) (2 \ 3)$

$$(5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4) = \\ (5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 4) (3 \ 4)$$

$$(6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5) = \\ (6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 5) (4 \ 5)$$

$$(7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \underline{6}) = \\ (7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 6) (\underline{5} \ 6)$$

$$(8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) = \\ (8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 7) (6 \ 7)$$

$$(9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) = \\ (9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 8) (7 \ 8)$$

$$(10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9) = \\ (10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 9) (8 \ 9)$$

$$\begin{aligned} & (11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ \underline{10}) = \\ & (11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ \underline{10}) (9 \ \underline{10}) \end{aligned}$$

$$(12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11) = \\ (12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 11) (10 \ 11)$$

$$(13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12) = \\ (13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 12) (11 \ 12)$$

$$(14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ \underline{13}) = \\ (14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 13) (12 \ 13)$$

$$(15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14) = \\ (15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 14) (13 \ 14)$$

$$(16 \ 17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15) = \\ (16 \ 17 \ 18 \ 19 \ 20 \ 12 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 15) \ (14 \ 15)$$

$$(17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16) = \\ (17 \ 18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 16) \ (15 \ 16)$$

$$(18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17) = \\ (18 \ 19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 17) \ (16 \ 17)$$

$$(19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18) = \\ (19 \ 20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 18) \ (17 \ 18)$$

$$(20 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19) = \\ (20, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19), (18, 19)$$

$$(22c) = (2c)(21c) - \text{Left}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 \end{pmatrix} = \\ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 \end{pmatrix}$$

$$(\underline{2} \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1) = \\ (\underline{2} \ 3) (\underline{4} \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1)$$

$$(3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2) = (3 \ 4) (3 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2)$$

$$(4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3) = \\ (4 \ 5) (4 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3)$$

$$(5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4) = \\ (5 \ 6) (7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4)$$

$$\begin{aligned} & (6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5) = \\ & (6 \ 7) (6 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5) \end{aligned}$$

$$(\underline{7} \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6) = \\ (\underline{7} \ 8) (\underline{9} \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6)$$

$$\begin{aligned} & (8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) = \\ & (8 \ 9) (8 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) \end{aligned}$$

$$\begin{pmatrix} 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{pmatrix} = \\ \begin{pmatrix} 9 & 10 \\ 9 & 10 \end{pmatrix} \begin{pmatrix} 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{pmatrix}$$

$$\begin{aligned} & (\underline{10} \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9) = \\ & (\underline{10} \ 11) (\underline{10} \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9) \end{aligned}$$

$$(\underline{11} \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10) = \\ (11 \ 12) (11 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10)$$

$$(\underline{12} \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11) = \\ (12 \ 13) (12 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11)$$

$$(\underline{13} \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12) = \\ (13 \ 14) (13 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12)$$

$$(\underline{14} \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13) = \\ (14 \ 15)(14 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13)$$

$$(\underline{15} \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14) = \\ (15 \ 16) (15 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14)$$

$$\begin{pmatrix} 16 & 17 & 18 & 19 & 20 & 21 & 22 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{pmatrix} = \\ \begin{pmatrix} 16 & 17 \\ 17 & 16 \end{pmatrix} \begin{pmatrix} 16 & 18 & 19 & 20 & 21 & 22 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{pmatrix}$$

$$(\underline{17} \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16) = \\ (17 \ 18)(17 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16)$$

$$\begin{pmatrix} 18 & 19 & 20 & 21 & 22 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 \end{pmatrix} = \\ (18 \ 19) (18 \ 20 \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17)$$

$$\begin{pmatrix} 19 & 20 & 21 & 22 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \end{pmatrix} = \\ \begin{pmatrix} 19 & 20 \\ 19 & 20 \end{pmatrix} \begin{pmatrix} 21 & 22 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \end{pmatrix}$$

$$(\underline{20} \ 21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19) = \\ (\underline{20}, 1)(20, 22, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19)$$

$$(21 \ 22 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20) = \\ (21 \ 22) (21 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20)$$

$$(\underline{22} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21) = \\ (\underline{22}, 1), (\underline{22}, 2), (\underline{22}, 3), (\underline{22}, 4), (\underline{22}, 5), (\underline{22}, 6), (\underline{22}, 7), (\underline{22}, 8), (\underline{22}, 9), (\underline{22}, 10), (\underline{22}, 11), (\underline{22}, 12), (\underline{22}, 13), (\underline{22}, 14), (\underline{22}, 15), (\underline{22}, 16), (\underline{22}, 17), (\underline{22}, 18), (\underline{22}, 19), (\underline{22}, 20), (\underline{22}, 21)$$

(22a) – (24a) (2a) Right

$$(1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22) = \\ (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 22) (21 \ 22)$$

$$(2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \underline{1}) = \\ (2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \underline{1}) (22 \underline{1})$$

$$(3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 1 \ 2) =$$

(3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 2) (1 2)

(4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3) =  
(4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 3) (2 3)

(5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4) =  
(5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 4) (3 4)

(6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5) =  
(6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 5) (4 5)

(7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6) =  
(7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 6) (5 6)

(8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7) =  
(8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 7) (6 7)

(9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8) =  
(9 10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 8) (7 8)

(10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9) =  
(10 11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 9) (8 9)

(11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10) =  
(11 12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 10) (9 10)

(12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11) =  
(12 13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 11) (10 11)

(13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12) =  
(13 14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 12) (11 12)

(14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13) =  
(14 15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 13) (12 13)

(15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14) =  
(15 16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 14) (13 14)

(16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15) =  
(16 17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 15) (14 15)

(17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16) =  
(17 18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 16) (15 16)

(18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17) =  
(18 19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 17) (16 17)

(19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18) =  
(19 20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 18) (17 18)

(20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19) =  
(20 21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 19) (18 19)

(21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20) =  
(21 22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 20) (19 20)

(22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21) =  
(22 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21) (20 21)

# Block Move Algorithms

## Block Move Algorithms – Four Moves

Corner-Centers		
Seed Algorithms	Cycle Structure	Number of Permuted Pieces
NR NU' NR' NU	(3c) (3c)	6
NR2 NU' NR2 NU	(3c) (3c)	6
NR2 NU2 NR2 NU2	(3c) (3c)	6

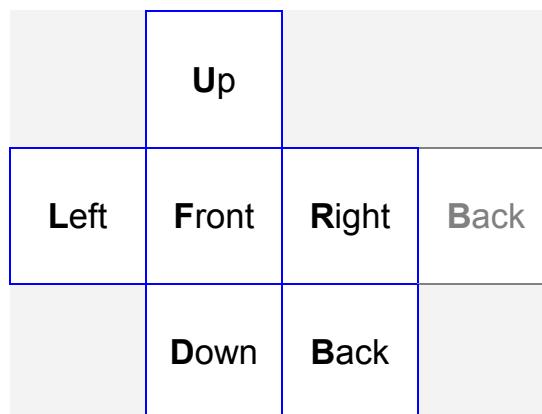
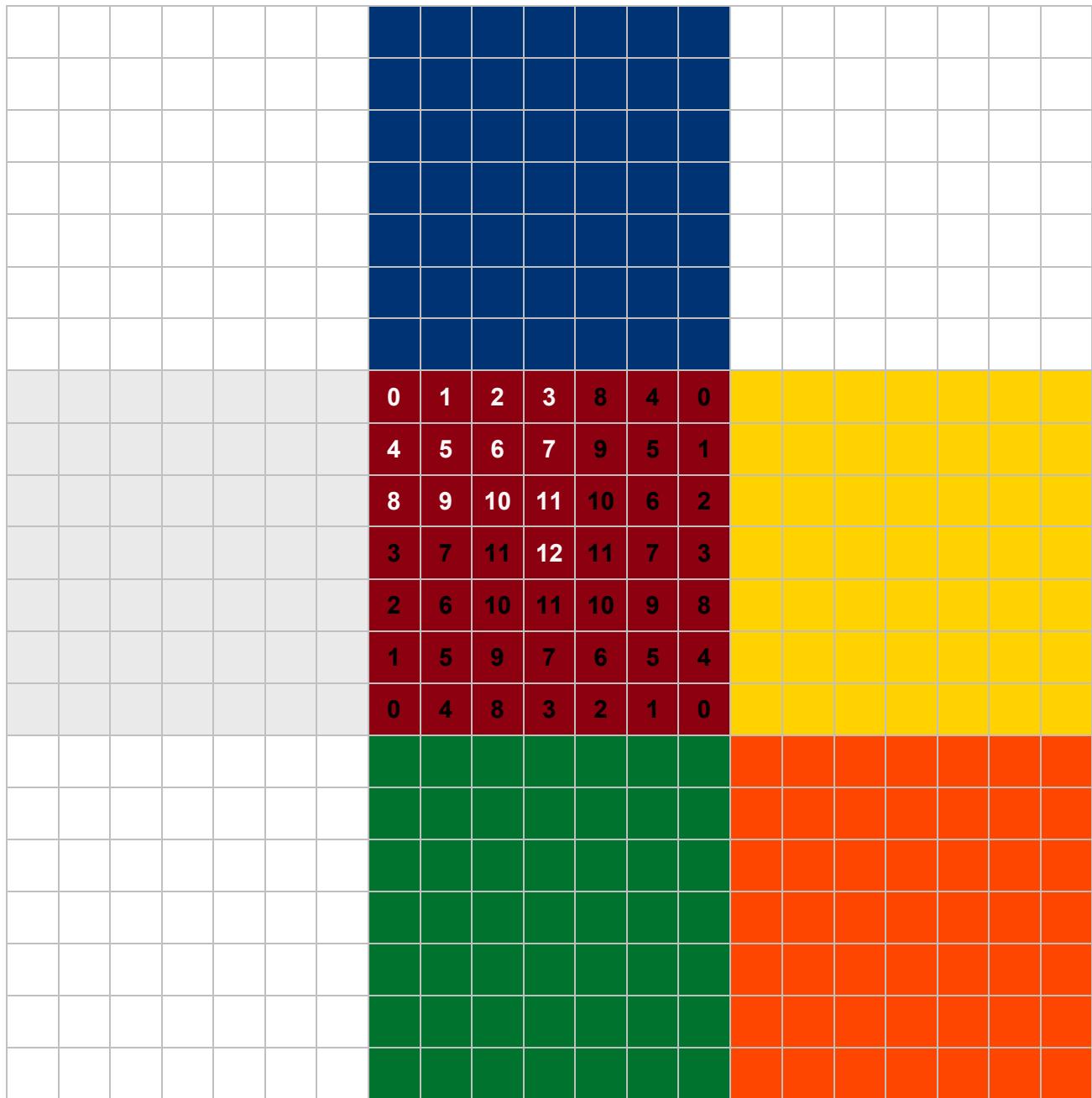
Midge-Centers		
Seed Algorithms	Cycle Structure	Number of Permuted Pieces
MR NU' MR' NU	(3c) (3c)	6
MR2 NU' MR2 NU	(3c) (3c)	6
MR NU2 MR' NU2	(2c) (2c)	4
MR2 NU2 MR2 NU2	(2c) (2c)	4

# Annex

## Cube Numbering System

### Orbit Numbering – 7x7x7 Cube

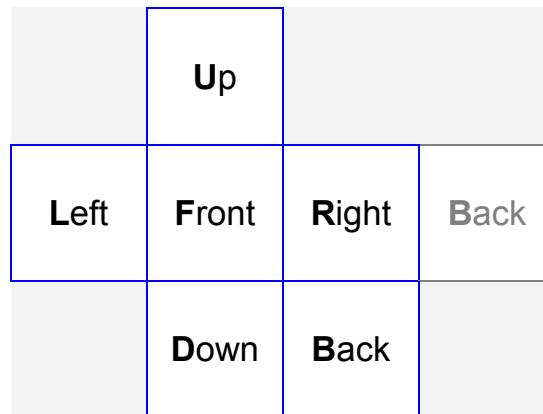
Orbits of stickers on a 7x7x7 cube are numbered from 0 up to 12, beginning at corners and ending up at true centers.



## Sticker Numbering – 7x7x7 Cube

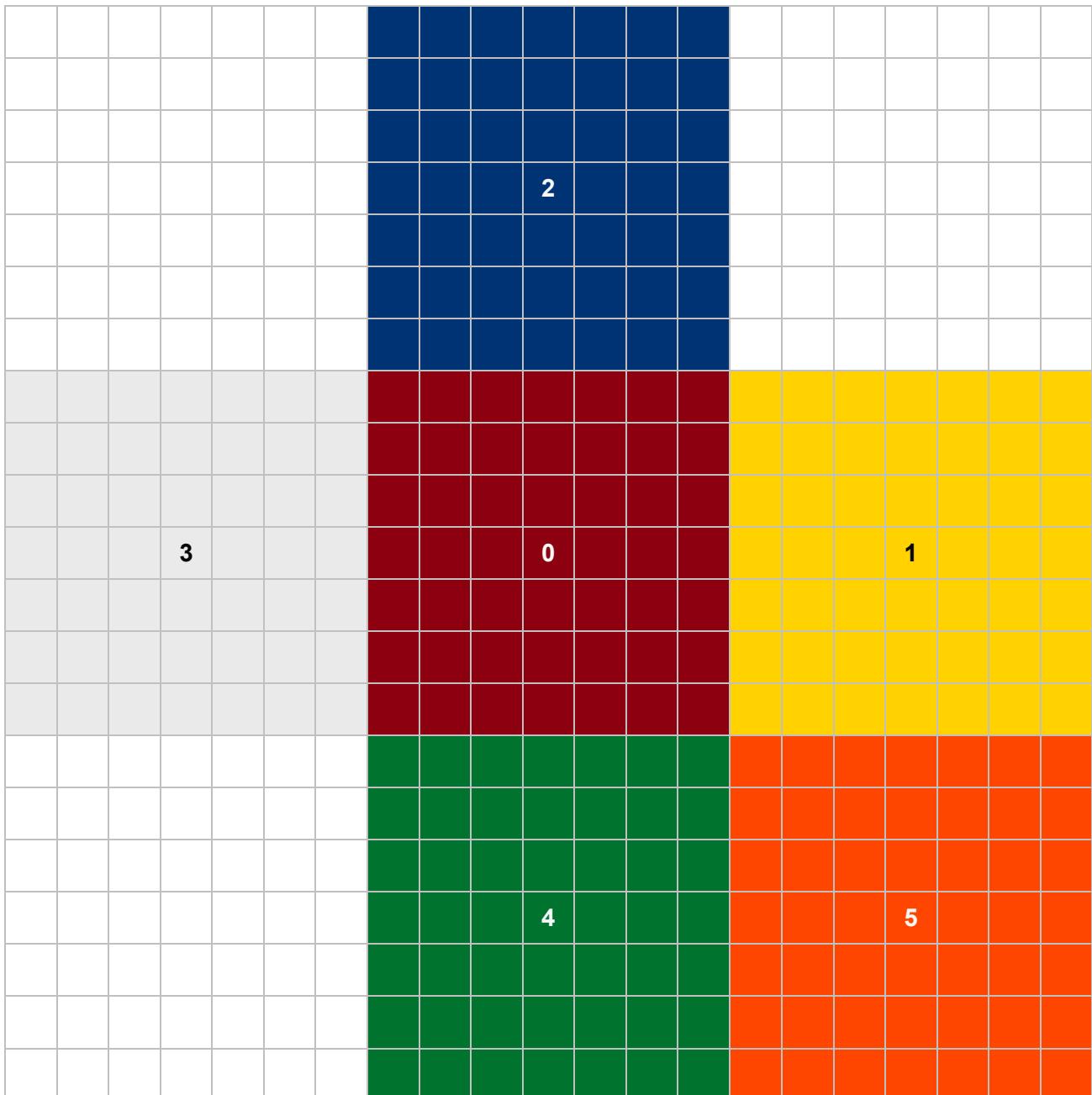
Stickers on a 7x7x7 cube are numbered *in rows* from 0 up to 293, beginning on face F, top left and ending on face B, bottom right, following sequence F R U L D B. Stickers can also be numbered *per orbit* from 0 up to 23.

								<b>98</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>									
								<b>105</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>									
								<b>112</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>									
								<b>119</b>	<b>120</b>	<b>121</b>	<b>122</b>	<b>123</b>	<b>124</b>	<b>125</b>									
								<b>126</b>	<b>127</b>	<b>128</b>	<b>129</b>	<b>130</b>	<b>131</b>	<b>132</b>									
								<b>133</b>	<b>134</b>	<b>135</b>	<b>136</b>	<b>137</b>	<b>138</b>	<b>139</b>									
								<b>140</b>	<b>141</b>	<b>142</b>	<b>143</b>	<b>144</b>	<b>145</b>	<b>146</b>									
<b>147</b>	<b>148</b>	<b>149</b>	<b>150</b>	<b>151</b>	<b>152</b>	<b>153</b>		<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>49</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>		
<b>154</b>	<b>155</b>	<b>156</b>	<b>157</b>	<b>158</b>	<b>159</b>	<b>160</b>		<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>56</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>		
<b>161</b>	<b>162</b>	<b>163</b>	<b>164</b>	<b>165</b>	<b>166</b>	<b>167</b>		<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>63</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>		
<b>168</b>	<b>169</b>	<b>170</b>	<b>171</b>	<b>172</b>	<b>173</b>	<b>174</b>		<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>70</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>		
<b>175</b>	<b>176</b>	<b>177</b>	<b>178</b>	<b>179</b>	<b>180</b>	<b>181</b>		<b>28</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>77</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>		
<b>182</b>	<b>183</b>	<b>184</b>	<b>185</b>	<b>186</b>	<b>187</b>	<b>188</b>		<b>35</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>84</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>		
<b>189</b>	<b>190</b>	<b>191</b>	<b>192</b>	<b>193</b>	<b>194</b>	<b>195</b>		<b>42</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>91</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>		
								<b>196</b>	<b>197</b>	<b>198</b>	<b>199</b>	<b>200</b>	<b>201</b>	<b>202</b>	<b>245</b>	<b>246</b>	<b>247</b>	<b>248</b>	<b>249</b>	<b>250</b>	<b>251</b>		
								<b>203</b>	<b>204</b>	<b>205</b>	<b>206</b>	<b>207</b>	<b>208</b>	<b>209</b>	<b>252</b>	<b>253</b>	<b>254</b>	<b>255</b>	<b>256</b>	<b>257</b>	<b>258</b>		
								<b>210</b>	<b>211</b>	<b>212</b>	<b>213</b>	<b>214</b>	<b>215</b>	<b>216</b>	<b>259</b>	<b>260</b>	<b>261</b>	<b>262</b>	<b>263</b>	<b>264</b>	<b>265</b>		
								<b>217</b>	<b>218</b>	<b>219</b>	<b>220</b>	<b>221</b>	<b>222</b>	<b>223</b>	<b>266</b>	<b>267</b>	<b>268</b>	<b>269</b>	<b>270</b>	<b>271</b>	<b>272</b>		
								<b>224</b>	<b>225</b>	<b>226</b>	<b>227</b>	<b>228</b>	<b>229</b>	<b>230</b>	<b>273</b>	<b>274</b>	<b>275</b>	<b>276</b>	<b>277</b>	<b>278</b>	<b>279</b>		
								<b>231</b>	<b>232</b>	<b>233</b>	<b>234</b>	<b>235</b>	<b>236</b>	<b>237</b>	<b>280</b>	<b>281</b>	<b>282</b>	<b>283</b>	<b>284</b>	<b>285</b>	<b>286</b>		
								<b>238</b>	<b>239</b>	<b>240</b>	<b>241</b>	<b>242</b>	<b>243</b>	<b>244</b>	<b>287</b>	<b>288</b>	<b>289</b>	<b>290</b>	<b>291</b>	<b>292</b>	<b>293</b>		



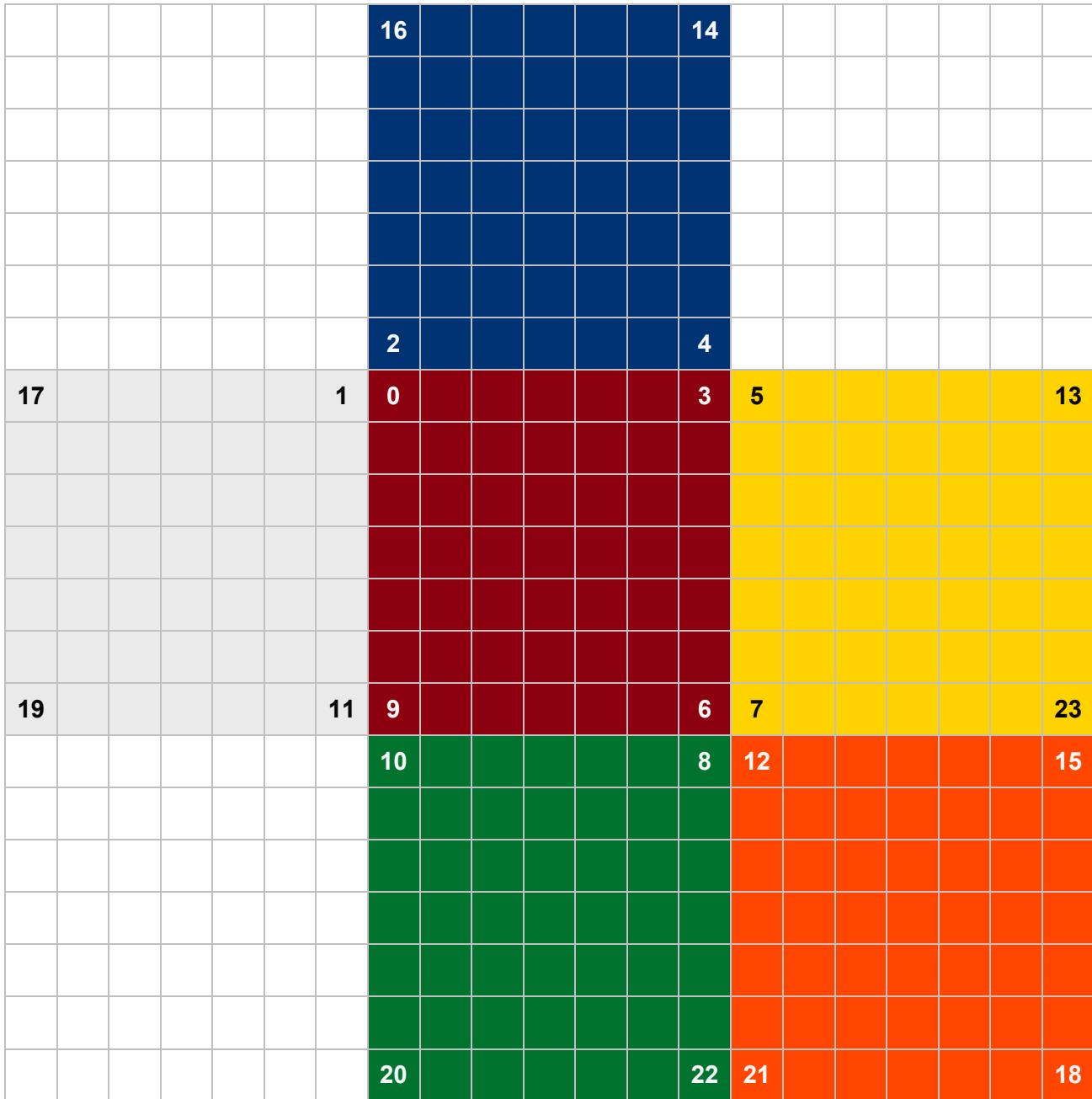
## True Center Orbit

The 6 numbered stickers of the orbit of true centers of a 7x7x7 cube are shown on the texture below:



## Corner Orbit – Stickers

The 24 numbered *stickers* of the corner orbit of a 7x7x7 cube are shown on the texture below:



Reference Cube								
Corner	0	1	2	3	4	5	6	7
Stickers	(0, 1, 2)	(3, 4, 5)	(6, 7, 8)	(9, 10, 11)	(12, 13, 14)	(15, 16, 17)	(18, 19, 20)	(21, 22, 23)
Orbit Cube								
Stickers	(D, O, F)	(A, E, I)	(B, L, S)	(C, R, P)	(W, J, H)	(X, G, N)	(U, M, Q)	(V, T, K)

Corner *stickers* are numbered from 0 to 23 whereas corner *pieces* are numbered from 0 to 7.

Sticker numbers have been selected such that a cyclic permutation of numbers in a group of 3 will always twist a corner in a *clockwise* direction, so that sticker permutation (0 → 1 → 2 → 0) will twist corner 0 clockwise (CW) whereas sticker permutation (0 → 2 → 1 → 0) will twist it counterclockwise (CCW).

For a group of 3 stickers ( $s_0, s_1, s_2$ ) located on a *same* corner piece, the corner number can be obtained from:

$$\text{corner number} = \text{Integer}(s_0 / 3), \text{ or Integer}(s_1 / 3), \text{ or Integer}(s_2 / 3)$$

Given a corner sticker number  $s_0$ , the 2 other corner sticker numbers can be obtained from:

$$s_0 = 3 * \text{Integer}(s_0 / 3) + (s_0 + 0) \text{ Modulo } 3$$

```
s1 = 3 * Integer (s0 / 3) + (s0 + 1) Modulo 3
s2 = 3 * Integer (s0 / 3) + (s0 + 2) Modulo 3
```

## Corner Twists

There are four permuted corners per face turn. Quarter-turns turns F, F', B, B' and all half-turns F2, R2, U2, L2, D2, B2 do not change the twist of any of the four permuted corners, whereas quarter-turns R, R', U, U', L, L', D, D' change the twist of only *half* of the permuted corners.

To obtain the new twist of a corner that has been moved to a new position, simply add the delta twist value indicated in the [Table of Corner Twists](#) to the old corner twist:

$$\text{newTwist} = (\text{oldTwist} + \text{deltaTwist}) \text{ Modulo } 3$$

For example, by applying move R to the right face, the new twist of the corner that has been moved from old position 4 to new position 7 is given by:

$$\begin{aligned} \text{deltaTwist} &= +1 \\ \text{newTwist}(7) &= (\text{oldTwist}(4) + 1) \text{ Modulo } 3 \end{aligned}$$

If this corner located at position 7 is now moved back to position 4 by move R', then the 2 delta twists will cancel out Modulo 3:

$$\begin{aligned} \text{deltaTwist} &= +2 \\ \text{newTwist}(4) &= (\text{oldTwist}(7) + 2) \text{ Modulo } 3 \\ \text{newTwist}(4) &= ((\text{oldTwist}(4) + 1) + 2) \text{ Modulo } 3 \\ \text{newTwist}(4) &= (\text{oldTwist}(4) + (1 + 2)) \text{ Modulo } 3 \\ \text{newTwist}(4) &= (\text{oldTwist}(4) + 3) \text{ Modulo } 3 \\ \text{newTwist}(4) &= \text{oldTwist}(4) \end{aligned}$$

The table shows that any *legal* face move will *always* keep the sum of \*all\* corner twists equal to zero Modulo 3.

**Table of Corner Twists**

**Face Move F**

old corner position	0	1	2	3	4	5	6	7
new corner position	1	2	3	0	4	5	6	7
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

**Face Move F2**

old corner position	0	1	2	3	4	5	6	7
new corner position	2	3	0	1	4	5	6	7
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

**Face Move F'**

old corner position	0	1	2	3	4	5	6	7
new corner position	3	0	1	2	4	5	6	7
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

**Face Move R**

old corner position	0	1	2	3	4	5	6	7
new corner position	0	4	1	3	7	5	6	2
deltaTwist	+0	+2	+1	+0	+1	+0	+0	+2

**Face Move R2**

old corner position	0	1	2	3	4	5	6	7
new corner position	0	7	4	3	2	5	6	1
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

**Face Move R'**

old corner position	0	1	2	3	4	5	6	7
new corner position	0	2	7	3	1	5	6	4
deltaTwist	+0	+2	+1	+0	+1	+0	+0	+2

**Face Move U**

old corner position	0	1	2	3	4	5	6	7
new corner position	5	0	2	3	1	4	6	7
deltaTwist	+2	+1	+0	+0	+2	+1	+0	+0

**Face Move U2**

old corner position	0	1	2	3	4	5	6	7
new corner position	4	5	2	3	0	1	6	7
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

**Face Move U'**

old corner position	0	1	2	3	4	5	6	7
new corner position	1	4	2	3	5	0	6	7
deltaTwist	+1	+2	+0	+0	+2	+1	+0	+0

**Face Move L**

old corner position	0	1	2	3	4	5	6	7
new corner position	3	1	2	6	4	0	5	7
deltaTwist	+1	+0	+0	+2	+0	+2	+1	+0

**Face Move L2**

old corner position	0	1	2	3	4	5	6	7
new corner position	6	1	2	5	4	3	0	7
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

**Face Move L'**

old corner position	0	1	2	3	4	5	6	7
new corner position	5	1	2	0	4	6	3	7
deltaTwist	+1	+0	+0	+2	+0	+2	+1	+0

**Face Move D**

old corner position	0	1	2	3	4	5	6	7
new corner position	0	1	7	2	4	5	3	6
deltaTwist	+0	+0	+2	+1	+0	+0	+2	+1

**Face Move D2**

old corner position	0	1	2	3	4	5	6	7
new corner position	0	1	6	7	4	5	2	3
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

**Face Move D'**

old corner position	0	1	2	3	4	5	6	7
new corner position	0	1	3	6	4	5	7	2
deltaTwist	+0	+0	+2	+1	+0	+0	+2	+1

**Face Move B**

old corner position	0	1	2	3	4	5	6	7
new corner position	0	1	2	3	5	6	7	4
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

**Face Move B2**

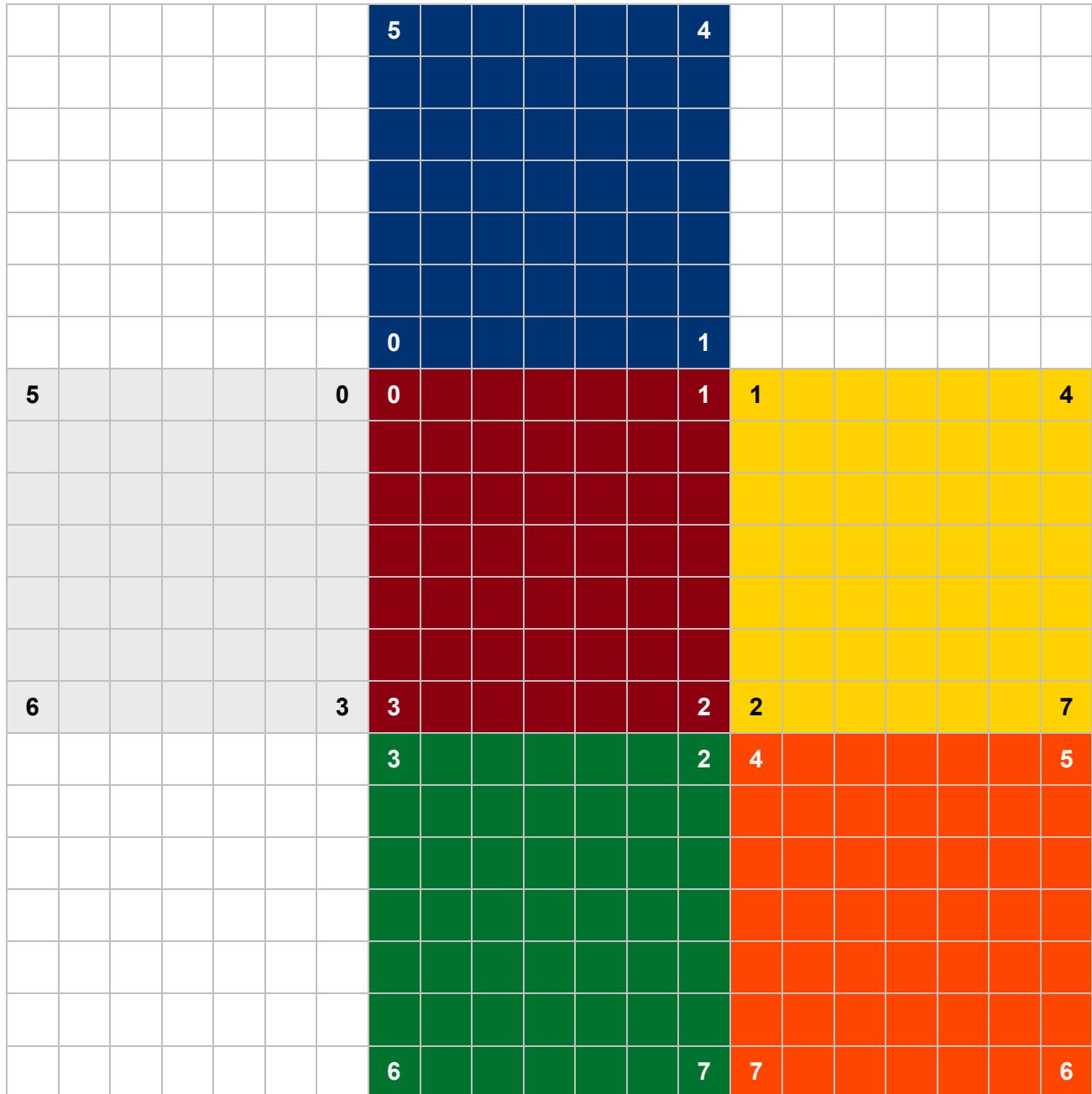
old corner position	0	1	2	3	4	5	6	7
new corner position	0	1	2	3	6	7	4	5
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

**Face Move B'**

old corner position	0	1	2	3	4	5	6	7
new corner position	0	1	2	3	7	4	5	6
deltaTwist	+0	+0	+0	+0	+0	+0	+0	+0

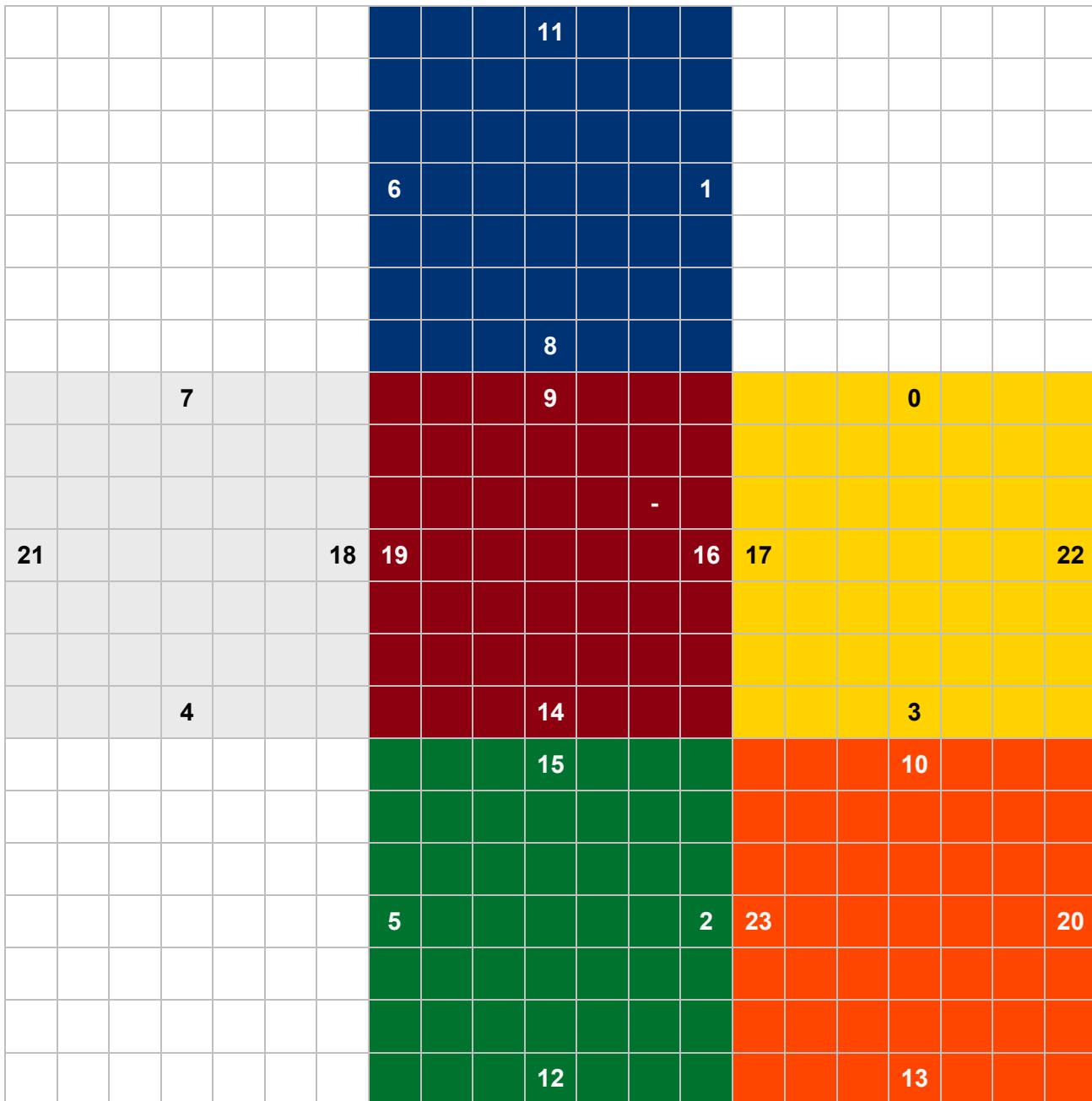
## Corner Orbit – Pieces

The 8 numbered *pieces* of the corner orbit of a 7x7x7 cube are shown on the texture below:



## Midge Orbit – Stickers

The 24 numbered *stickers* of the midge orbit of a 7x7x7 cube are shown on the texture below:



Reference Cube												
Midge	0	1	2	3	4	5	6	7	8	9	10	11
Stickers	(0, 1)	(2, 3)	(4, 5)	(6, 7)	(8, 9)	(10, 11)	(12, 13)	(14, 15)	(16, 17)	(18, 19)	(20, 21)	(22, 23)
Orbit Cube												
Stickers	(I, H)	(S, K)	(P, Q)	(F, N)	(E, D)	(W, G)	(T, U)	(B, R)	(A, L)	(O, C)	(X, M)	(J, V)

Midge *stickers* are numbered from 0 to 23 whereas midge *pieces* are numbered from 0 to 11.

For a pair of 2 stickers ( $s_0, s_1$ ) located on a *same* midge piece, the midge piece number can be obtained as:

$$\text{midge piece number} = \text{Integer}(s_0 / 2) \text{ or } \text{Integer}(s_1 / 2)$$

Given a midge sticker number  $s_0$ , the number  $s_1$  of the sticker located on the other side can be obtained from:

$$s_0 = 2 * \text{Integer}(s_0 / 2) + (s_0 + 0) \text{ Modulo } 2$$

$$s_1 = 2 * \text{Integer}(s_0 / 2) + (s_0 + 1) \text{ Modulo } 2$$

Given a midge sticker number  $s$  that has moved to sticker *position*  $p$ , the midge flip can be obtained from:

- 1- Compute  $S = s - 2 * \text{Integer}(s / 2)$
- 2- Compute  $P = p - 2 * \text{Integer}(p / 2)$
- 3- Compare  $S$  and  $P$  and compute midge flip from:

if ( $P = S$ ) then Flip = 0, else Flip = 1

## Midge Flips

There are four permuted midges per face turn. Half-turns F2, R2, U2, L2, D2, B2 toggle the flip of each of the four permuted midges, whereas quarter-turns F, F', R, R', U, U', L, L', D, D', B, B' toggle the flip of only *half* of the permuted midges.

To obtain the new flip of a midge that has been moved to a new position, simply add the delta flip value indicated in the [Table of Midge Flips](#) to the old midge flip:

$$\text{newFlip} = (\text{oldFlip} + \text{deltaFlip}) \text{ Modulo } 2$$

For example, by applying move R to the right face, the new flip of the midge that has been moved from old position 8 to new position 0 is given by:

$$\begin{aligned}\text{deltaFlip} &= +1 \\ \text{newFlip}(0) &= (\text{oldFlip}(8) + 1) \text{ Modulo } 2\end{aligned}$$

If this midge located at position 0 is now moved back to position 8 by move R', then the 2 delta flips will cancel out Modulo 2:

$$\begin{aligned}\text{deltaFlip} &= +1 \\ \text{newFlip}(8) &= (\text{oldFlip}(0) + 1) \text{ Modulo } 2 \\ \text{newFlip}(8) &= ((\text{oldFlip}(8) + 1) + 1) \text{ Modulo } 2 \\ \text{newFlip}(8) &= (\text{oldFlip}(8) + (1 + 1)) \text{ Modulo } 2 \\ \text{newFlip}(8) &= (\text{oldFlip}(8) + 2) \text{ Modulo } 2 \\ \text{newFlip}(8) &= \text{oldFlip}(8)\end{aligned}$$

The table shows that any *legal* face move will *always* keep the sum of \*all\* midge flips equal to zero Modulo 2.

Table of Midge Flips												
Face Move F												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	1	2	3	8	5	6	9	7	4	10	11
deltaFlip	+0	+0	+0	+0	+1	+0	+0	+1	+0	+0	+0	+0
Face Move F2												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	1	2	3	7	5	6	4	9	8	10	11
deltaFlip	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
Face Move F'												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	1	2	3	9	5	6	8	4	7	10	11
deltaFlip	+0	+0	+0	+0	+0	+0	+0	+0	+1	+1	+0	+0

Face Move R												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	11	8	2	3	4	5	6	7	0	9	10	1
deltaFlip	+0	+0	+0	+0	+0	+0	+0	+0	+1	+0	+0	+1
Face Move R2												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	1	0	2	3	4	5	6	7	11	9	10	8
deltaFlip	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
Face Move R'												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	8	11	2	3	4	5	6	7	1	9	10	0
deltaFlip	+1	+1	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0

### Face Move U

old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	4	1	2	5	3	0	6	7	8	9	10	11
deltaFlip	+1	+0	+0	+1	+0	+0	+0	+0	+0	+0	+0	+0
<b>Face Move U2</b>												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	3	1	2	0	5	4	6	7	8	9	10	11
deltaFlip	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
<b>Face Move U'</b>												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	5	1	2	4	0	3	6	7	8	9	10	11
deltaFlip	+0	+0	+0	+0	+1	+1	+0	+0	+0	+0	+0	+0

### Face Move L

old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	1	10	9	4	5	6	7	8	2	3	11
deltaFlip	+0	+0	+1	+1	+0	+0	+0	+0	+0	+0	+0	+0
<b>Face Move L2</b>												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	1	3	2	4	5	6	7	8	10	9	11
deltaFlip	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
<b>Face Move L'</b>												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	1	9	10	4	5	6	7	8	3	2	11
deltaFlip	+0	+0	+0	+0	+0	+0	+0	+0	+0	+1	+1	+0

### Face Move D

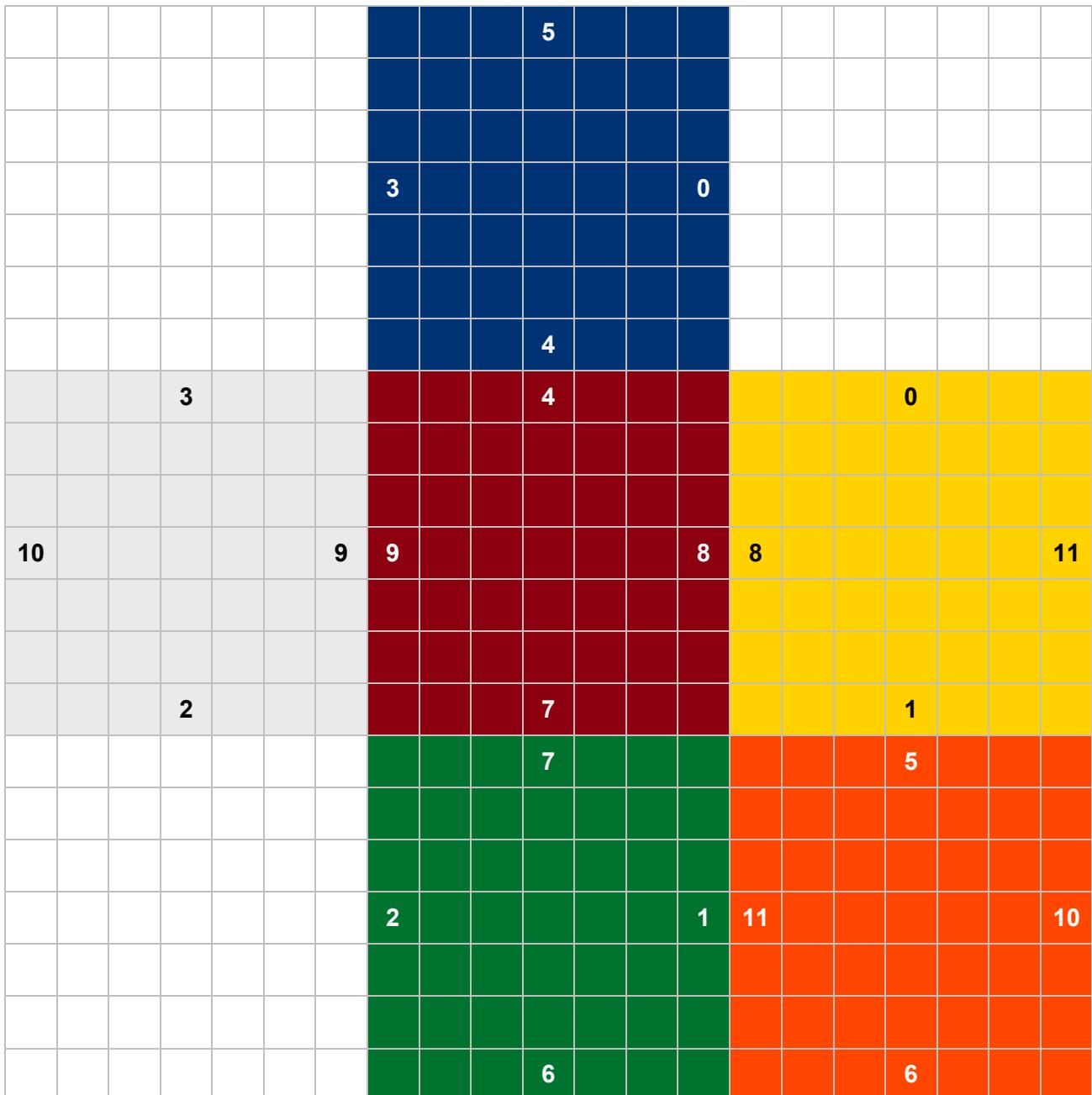
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	6	7	3	4	5	2	1	8	9	10	11
deltaFlip	+0	+0	+0	+0	+0	+0	+1	+1	+0	+0	+0	+0
<b>Face Move D2</b>												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	2	1	3	4	5	7	6	8	9	10	11
deltaFlip	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
<b>Face Move D'</b>												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	7	6	3	4	5	1	2	8	9	10	11
deltaFlip	+0	+1	+1	+0	+0	+0	+0	+0	+0	+0	+0	+0

### Face Move B

old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	1	2	3	4	10	11	7	8	9	6	5
deltaFlip	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+1	+1
<b>Face Move B2</b>												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	1	2	3	4	6	5	7	8	9	11	10
deltaFlip	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
<b>Face Move B'</b>												
old midge position	0	1	2	3	4	5	6	7	8	9	10	11
new midge position	0	1	2	3	4	11	10	7	8	9	5	6
deltaFlip	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+1	+1

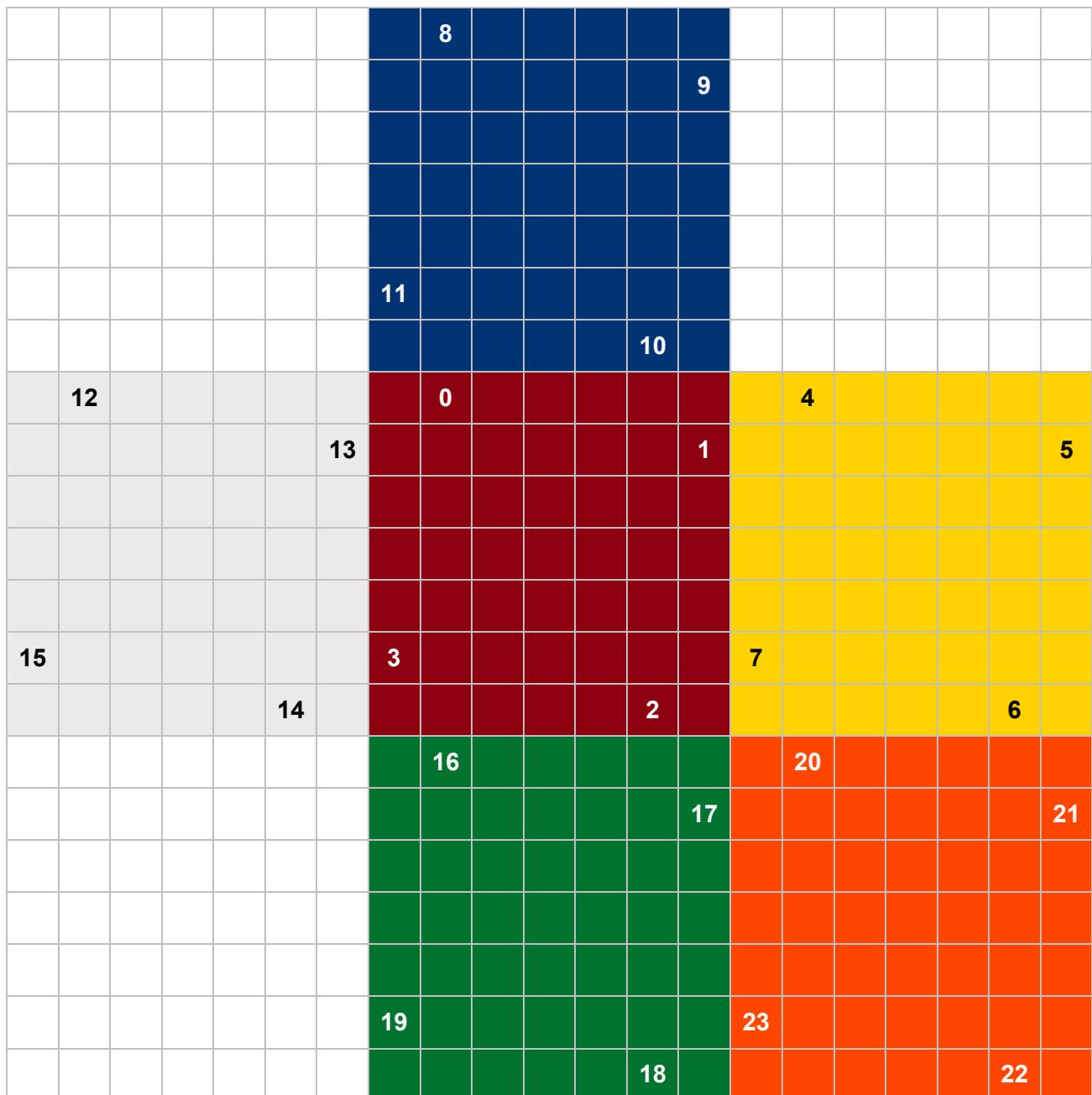
## Midge Orbit – Pieces

The 12 numbered *pieces* of the midge orbit of a 7x7x7 cube are shown on the texture below:

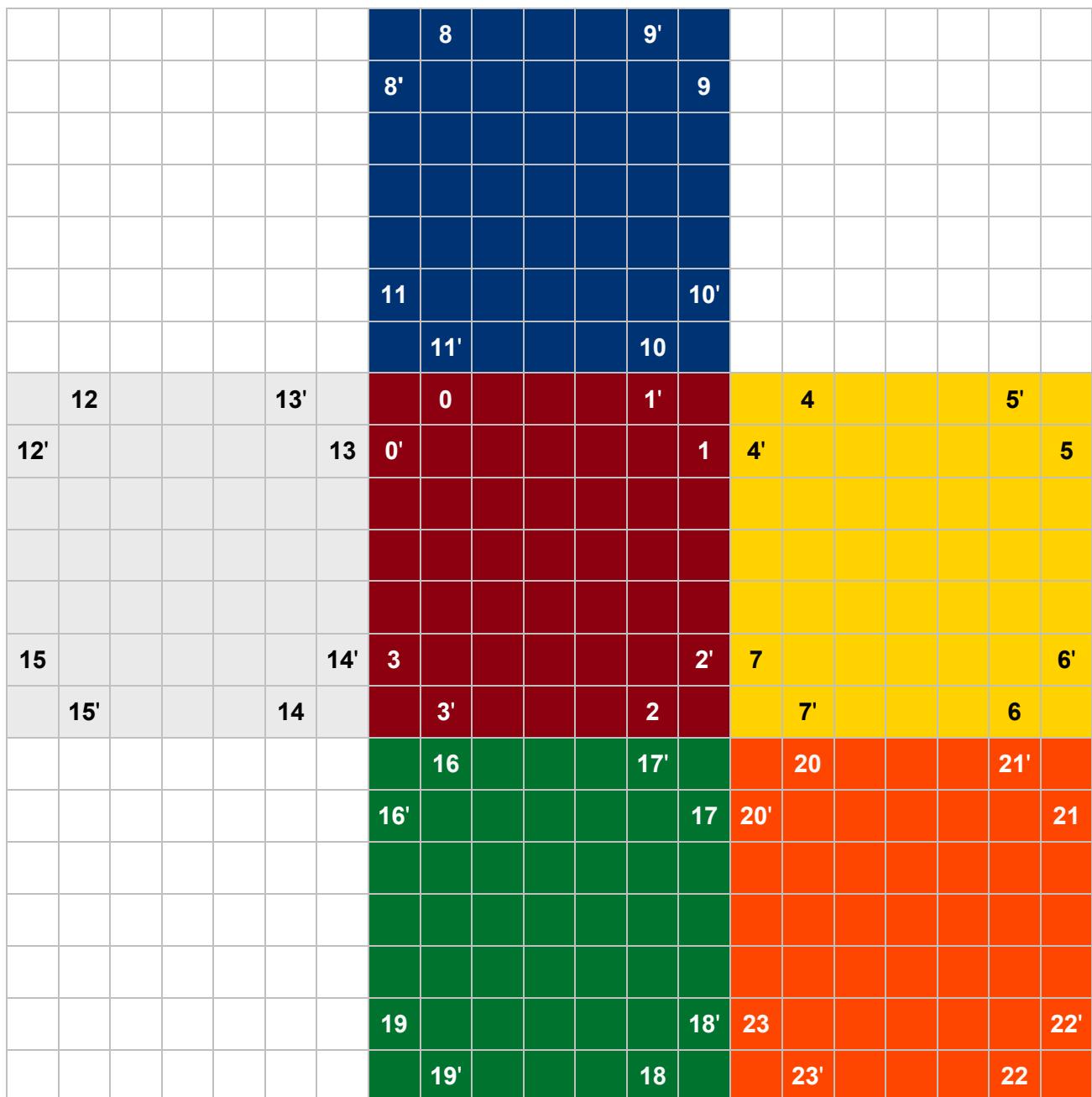


## Edge Orbit 1

The 24 numbered *stickers* of the 1<sup>st</sup> edge orbit of a 7x7x7 cube are shown on the texture below (one edge side):



The numbered stickers of the 1<sup>st</sup> edge orbit of a 7x7x7 cube are shown on the texture below (two edge sides).



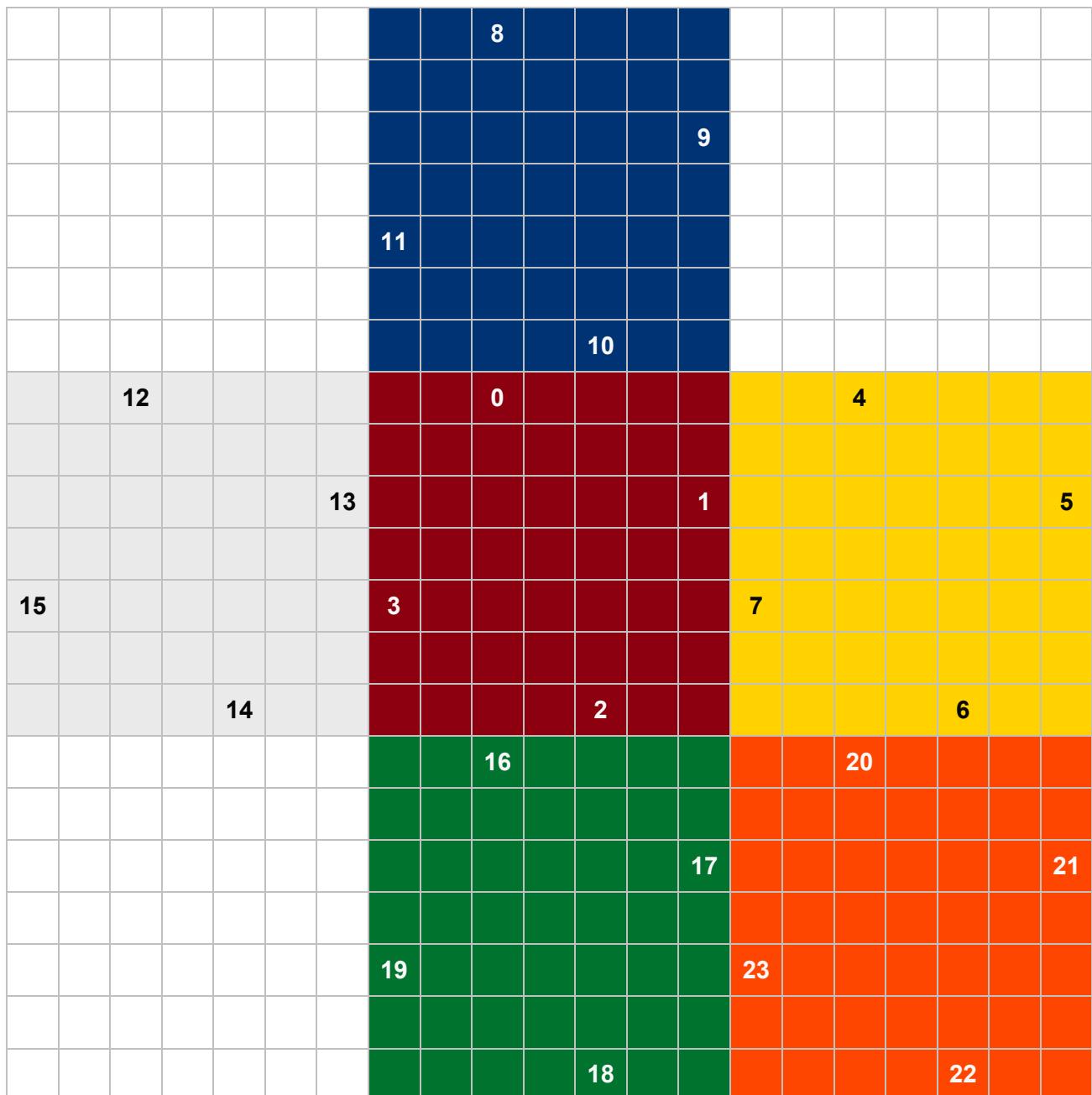
It can be seen that stickers of the two sides of an edge are paired. If an algorithm exists for cycling stickers of a side of an edge, then the same algorithm will also cycle stickers of the other side.

#### Edge Orbit 1 – Paired Stickers

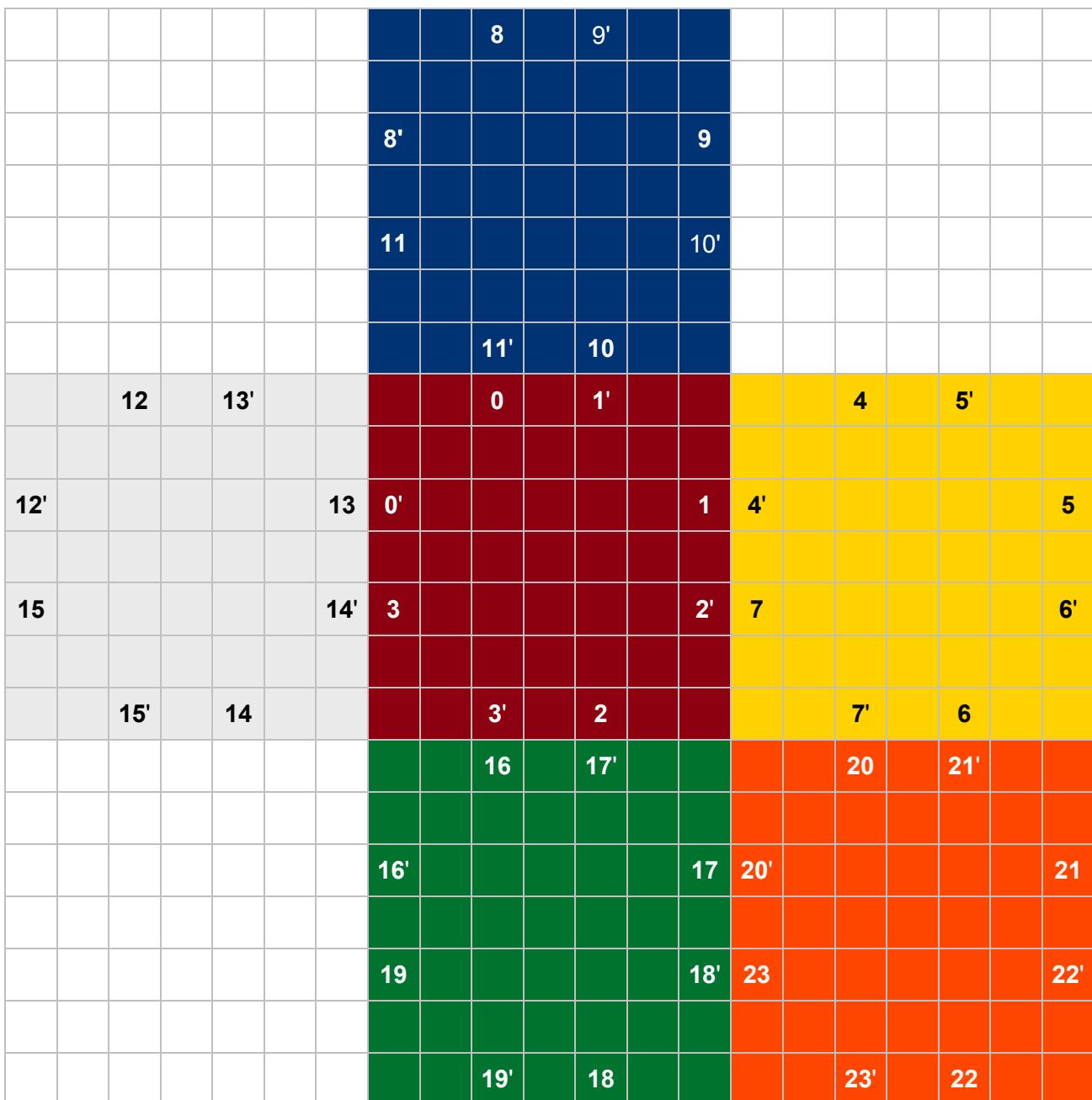
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
11'	4'	17'	14'	10'	20'	18'	2'	21'	5'	1'	13'	8'	0'	16'	22'	3'	7'	23'	15'	9'	12'	19'	6'
0'	1'	2'	3'	4'	5'	6'	7'	8'	9'	10'	11'	12'	13'	14'	15'	16'	17'	18'	19'	20'	21'	22'	23'
13	10	7	16	1	9	23	17	12	20	4	0	21	11	3	19	14	2	6	22	5	8	15	18

## Edge Orbit 2

The 24 numbered stickers of the 2<sup>nd</sup> edge orbit of a 7x7x7 cube are shown on the texture below (one edge side):



The numbered stickers of the 2<sup>nd</sup> edge orbit of a 7x7x7 cube are shown on the texture below (two edge sides):



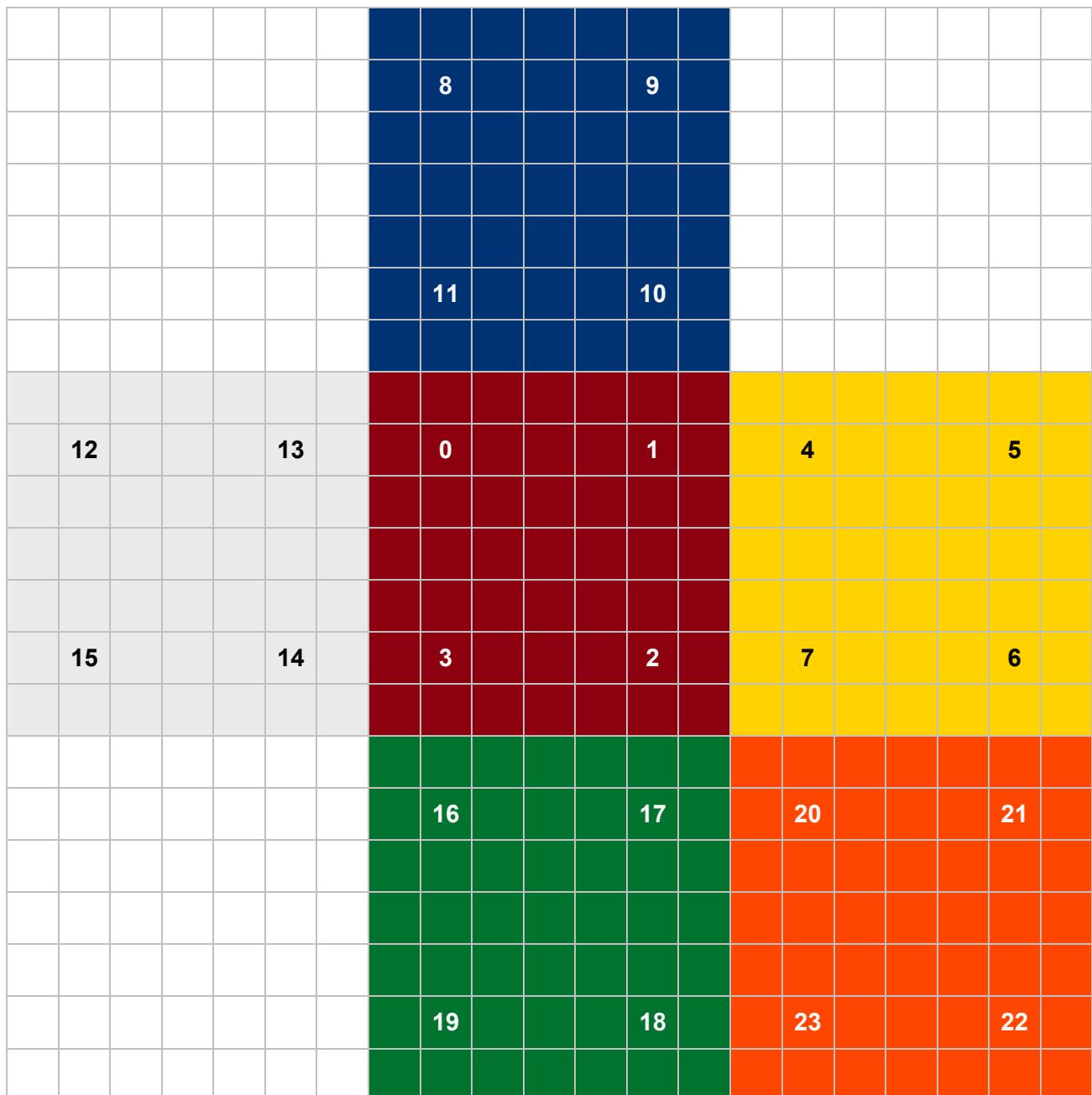
It can be seen that stickers of the two sides of an edge are paired. If an algorithm exists for cycling stickers of a side of an edge, then the same algorithm will also cycle stickers of the other side.

#### Edge Orbit 2 – Paired Stickers

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
11'	4'	17'	14'	10'	20'	18'	2'	21'	5'	1'	13'	8'	0'	16'	22'	3'	7'	23'	15'	9'	12'	19'	6'
0'	1'	2'	3'	4'	5'	6'	7'	8'	9'	10'	11'	12'	13'	14'	15'	16'	17'	18'	19'	20'	21'	22'	23'
13	10	7	16	1	9	23	17	12	20	4	0	21	11	3	19	14	2	6	22	5	8	15	18

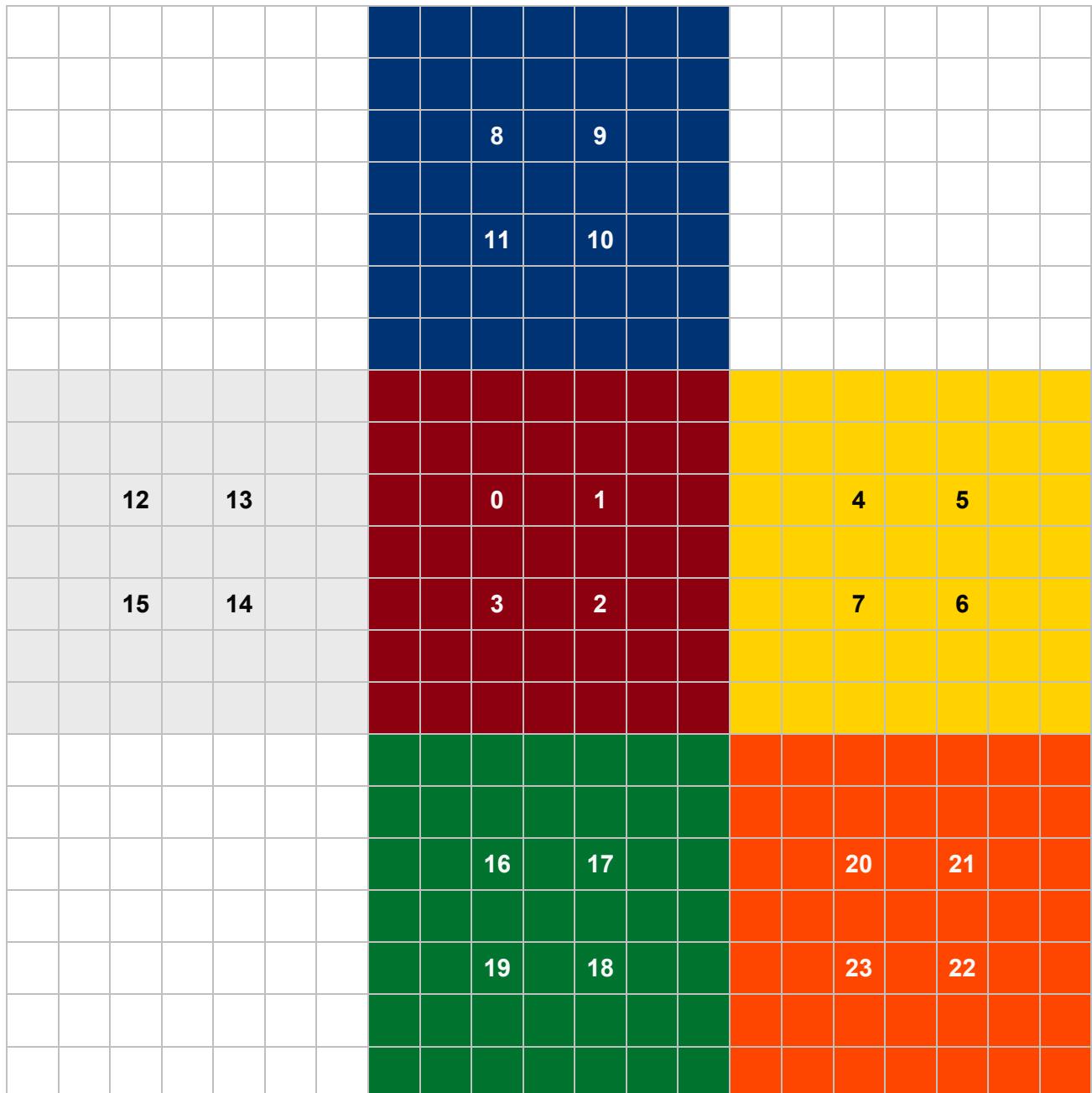
## Corner-Center Orbit 1

The 24 numbered stickers of the 1<sup>st</sup> corner-center orbit of a 7x7x7 cube are shown on the texture below:



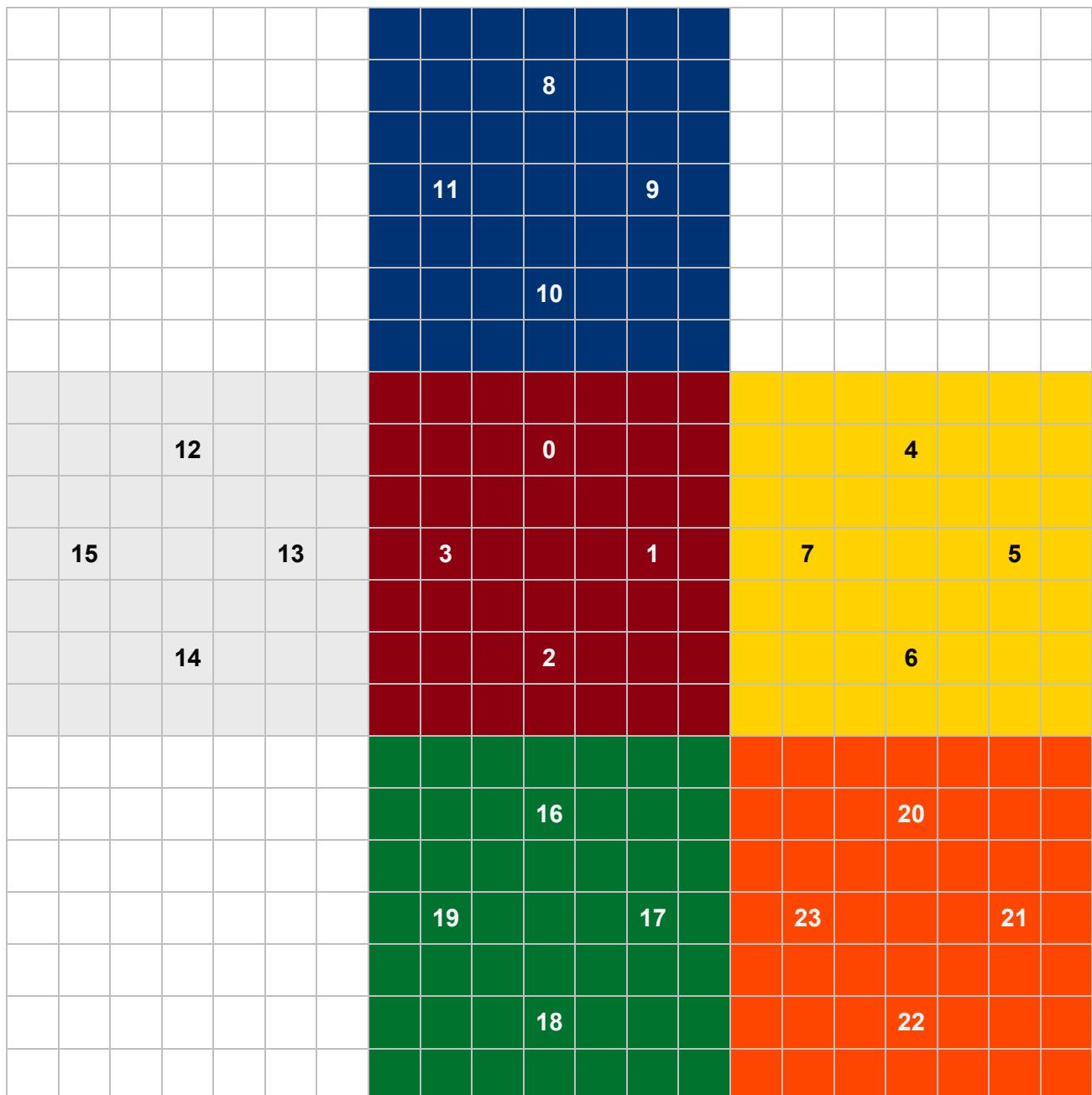
## Corner-Center Orbit 2

The 24 numbered stickers of the 2<sup>nd</sup> corner-center orbit of a 7x7x7 cube are shown on the texture below:



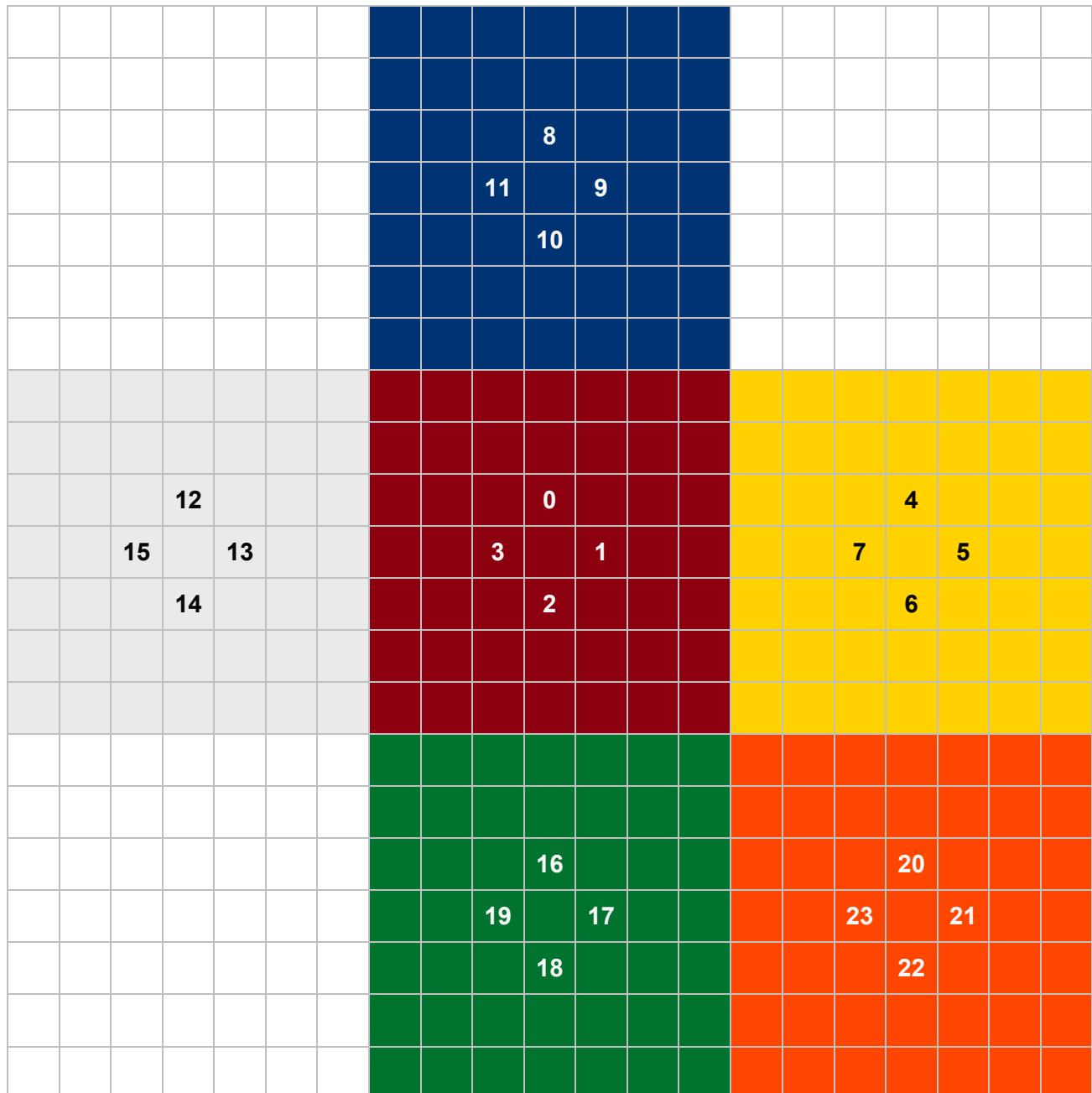
## Midge-Center Orbit 1

The 24 numbered stickers of the 1<sup>st</sup> midge-center orbit of a 7x7x7 cube are shown on the texture below:



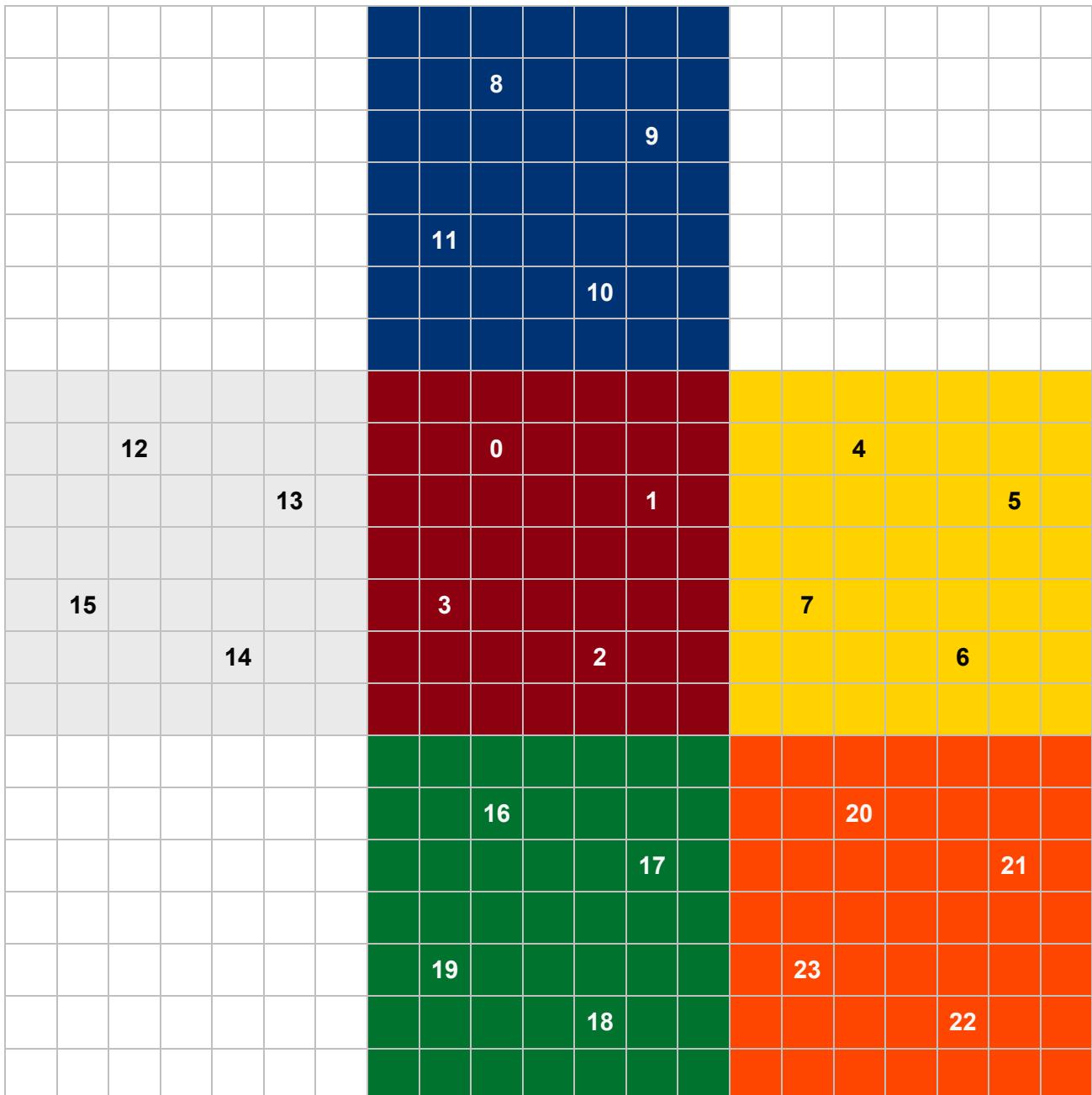
## Midge-Center Orbit 2

The 24 numbered stickers of the 2<sup>nd</sup> midge-center orbit of a 7x7x7 cube are shown on the texture below:



## Edge-Center Orbit 1

The 24 numbered stickers of the 1<sup>st</sup> edge-center orbit of a 7x7x7 cube are shown on the texture below:



## Edge-Center Orbit 2

The 24 numbered stickers of the 2<sup>nd</sup> edge-center orbit of a 7x7x7 cube are shown on the texture below:

