Mathematical Functions Cube Design

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Introduction

A **Mathematical Functions Cube** is a 3x3x3 **Rubik's Cube** used to display the argument and return values of some *selected* mathematical functions.

There are **Virtual Cubes** that can be *virtually* rotated and twisted on a computer screen and **Real Cubes** that can only be *physically* rotated and twisted by hand. A **Texture** is laid down on a Virtual Cube whereas real **Stickers** are stuck down on a Real Cube. A Mathematical Functions Cube is designed by placing letters and numbers on a texture which is then laid down on a Virtual Cube (see http://www.randelshofer.ch/ for more details).

Both argument and return value of a mathematical function can be displayed on a *selected* Cube face by rotating and twisting some parts of the Cube. When this has been achieved, we say that the Cube has been *solved*.

The result of *y=log(x)* with *x=1* and *y=.0* is shown on the following example:

Virtual Mathematical Functions Cube

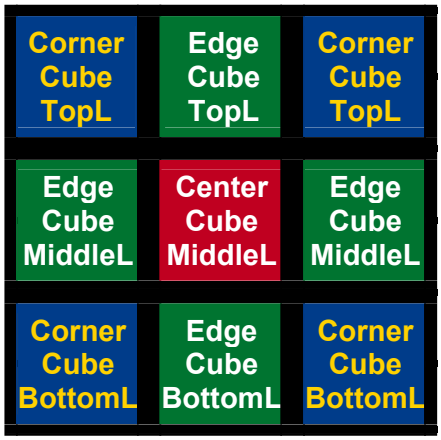
Mathematical Functions Cube Texture | Mathematical Functions Cube

Download **CubeTwister** from: http://www.randelshofer.ch/

Designing a Mathematical Functions Cube that *works* is definitely not a trivial task but **Design Rules** exist that should be applied. Because it is nearly impossible to test all configurations, the placement of numbers and letters on a texture should be carefully checked at *the end* of the design process. This is carried out by applying a **Design Rules Check** (**DRC**) in the final design stage.

Terminology

In a 3x3x3 **Rubik's Cube**, there are 8 *Corner Cubes*, 12 *Edge Cubes*, 6 *Center Cubes* and 6 *Cube Faces*. There are also 4 Corner Cube faces, 4 Edge Cube faces and 1 Center Cube face *per Cube Face*, as shown below.



There are 1 face per Center Cube, 2 faces per Edge Cube and 3 faces per Corner Cube.

There are also 3 horizontal *Layers* called *Top*, *Middle* and *Bottom Layers*.

Cube Lexicon		
English	**Français**	**Deutsch**
Cube	Cube	Würfel
cubie, cube	cube, petit cube	Würfelteil, Teil des Würfels
face	face	Seite, Seitenfläche
front face	face avant	vordere Seite, vorne
back face	face arrière	hintere Seite, hinten
left face	face gauche	linke Seite, links
right face	face droite	rechte Seite, rechts
top face	face supérieure	obere Seite, oben
bottom face	face inférieure	untere Seite, unten
sticker	étiquette (autocollante), plaquette	Kleber, Farbkleber
tile	tuile, plaquette	Plättchen, Farbplättchen
center cube, center	cube central, centre	Mittelwürfel, Mittelstein, Mitte
edge cube, edge	cube-arête, arête	Kantenwürfel, Kantenstein, Kante
corner cube, corner	cube de coin, coin	Eckwürfel, Eckstein, Ecke
layer	couronne	Schicht, Scheibe
top layer	couronne supérieure	obere Schicht, obere Scheibe
middle layer	couronne intermédiaire	mittlere Schicht, mittlere Scheibe, Mittelschicht, Mittelscheibe
bottom layer	couronne inférieure	untere Schicht, untere Scheibe
orientation, direction	orientation	Orientierung
to solve	résoudre	lösen, zusammen drehen
to twist	pivoter	drehen
to rotate	tourner, effectuer une rotation	drehen
clockwise	dans le sens horaire	im Uhrzeigersinn
anticlockwise, counter-clockwise	dans le sens anti-horaire	im Gegenuhrzeigersinn

Mathematical Functions Cube Design

In this *particular* design, the following 5 mathematical functions can be displayed on a cube, with *both* their argument and return values:

- LOG: base-10 logarithm function
- EXP: exponential function
- TAN: tangent function (argument value in degrees)
- SIN: sine function (argument value in degrees)
- COS: cosine function (argument value in degrees)

The cube can also be used to display return values of *inverse* functions. In that case, the value displayed on the bottom layer is the input value while the integer value on the middle layer is the output value. Inverse functions are shown on the following table:

Mathematical Functions and their Inverse			
Functions	Inverse Functions	Examples	
LOG	10^	LOG(10)=1	10^(1)=10
EXP	LN	EXP(1)=2.7	LN(2.7)=1
TAN	ATAN	TAN(45)=1	ATAN(1)=45
SIN	ASIN	SIN(30)=.5	ASIN(.5)=30
COS	ACOS	COS(45)=.7	ACOS(.7)=45

Cube Layout

After many attempts, the best layout that I have designed is as follows:

1- Function name on the top layer
2- Argument value on the middle layer
3- Return value on the bottom layer

Top Layer Layout

The function name should be displayed on the top layer. Letters are sorted out as follows:

1- 5 left-hand letters: L, E, T, ←S, C
2- 4 center letters: O, X, A, I
3- 4 right-hand letters: G, P, N, S→

Note 1: the 2 center letters of LOG and COS are the same, so only one 'O' is needed.
Note 2: the 2 final letters of TAN and SIN are the same, so only one 'N' is needed.
Note 3: the *intial* (←S) and *final* (S→) 'S' of SIN and COS should not be confused (**DRC**).

Letters are now combined on edge and corner cubes:

1- 2 edge cubes: (O,X), (A,I)
2- 3 corner cubes: (L,E,T), (S→,P,N), (G,←S,C)

Note 4: there are no groups of 2 letters of the same function name on the same corner cube (**DRC**).

So, now there are 10 edge and 5 corner cubes left that can be used for the 2 remaining layers.

Middle Layer Layout

The argument value should be displayed on the middle layer. Numbers are sorted out as follows:

1- Numbers on 6 center cubes: 0, 1, 2, 3, 4, 6 (or 9)
2- Left-hand numbers on 3 edge cubes: (0,1), (2,3), (4,white)
3- Right-hand numbers on 2 edge cubes: (5,7), (8,white)

Middle Layer		
0	0	
0	1	
0	2	
0	3	
0	4	
	0	5
0	6	
	0	7
	0	8

Middle Layer (cont'd)

0	9	
1	0	
	1	5
2	0	
	2	5
3	0	
	3	5
4	0	
4	1	
4	2	
4	3	
4	4	
	4	5
4	6	

Middle Layer (cont'd)		
	4	7
	4	8
4	9	

Argument values are displayed as 2-digit numbers. The argument value can then vary from 00 up to 49 by 01 steps. Numbers that end with 0, 1, 2, 3, 4, 6, 9 are displayed on the *left* while numbers that end with 5, 7, 8 are displayed on the *right*.

So, now there are 5 edge and 5 corner cubes left that can be used for the bottom layer.

Bottom Layer Layout

The function return value should be displayed on the bottom layer. Numbers are sorted out as follows:

1- 1 corner cube with (2,3,white), where '2' and '3' are left-hand numbers
2- 1 corner cube with (0,1,white), where '0' is on the *right* and '1' on the left
3- 3 corner cubes with all numbers on the right: (1,2,3), (4,5,6), (7,8,9)
4- 5 edge cubes: (.,0), (1,2), (3,4), (5,6), (7,8)

Bottom Layer					
Without decimal point			With decimal point		
	0	0		.	0
	0	1		.	1
	0	2		.	2
	0	3		.	3
	0	4		.	4
	0	5		.	5
	0	6		.	6

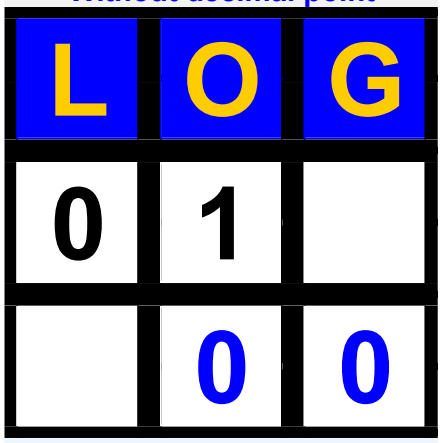
Bottom Layer (cont'd)					
Without decimal point			With decimal point		
	0	7		.	7
	0	8		.	8
	0	9		.	9
	1	0	1	.	0
	8	9	3	.	9

Return values can be displayed with a decimal point for values between .0 and 3.9 by .1 steps or as integer numbers from 00 to 89 by 01 steps.

Mathematical Functions Examples

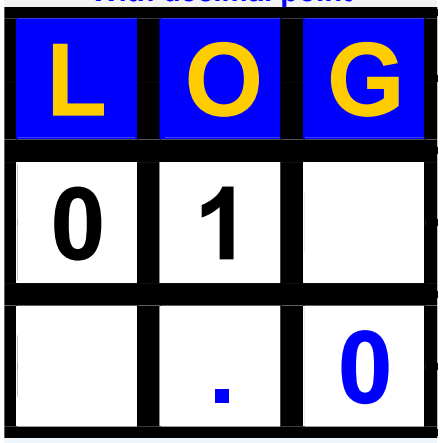
Base-10 Logarithmic Function

Without decimal point



LOG(1)=0

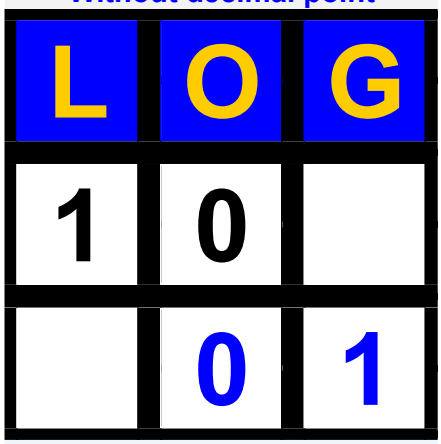
With decimal point



LOG(1)=0

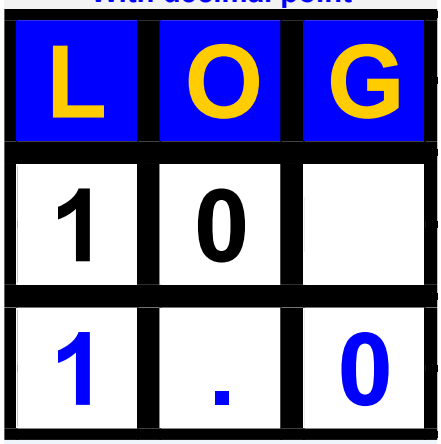
Base-10 Logarithmic Function

Without decimal point



LOG(10)=1

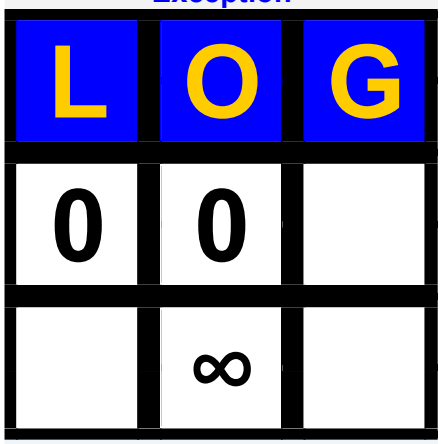
With decimal point



LOG(10)=1

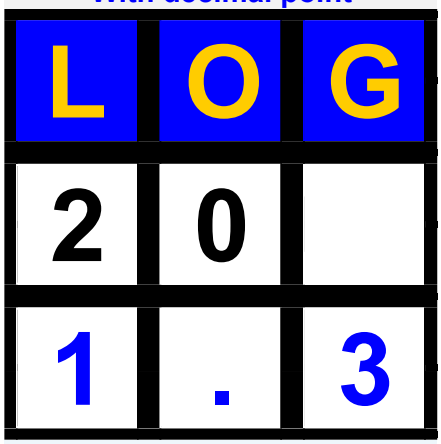
Base-10 Logarithmic Function

Exception



LOG(0)=∞

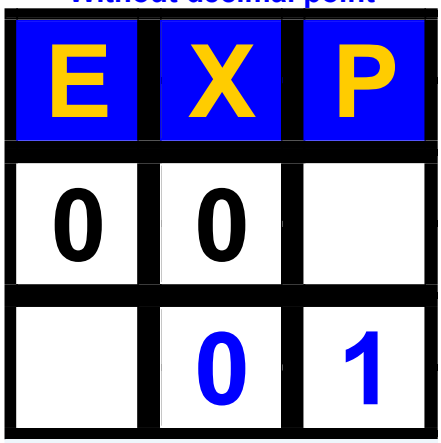
With decimal point



LOG(20)=1.3

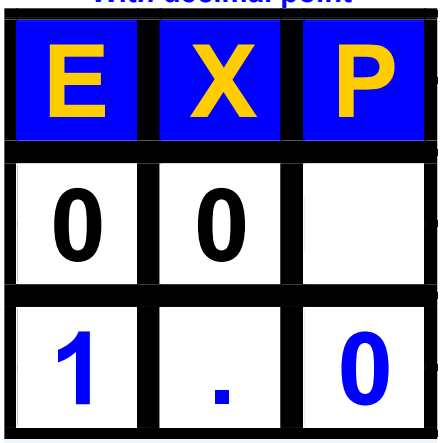
Exponential Function

Without decimal point



EXP(0)=1

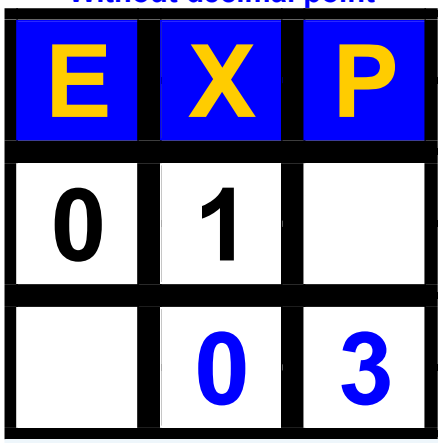
With decimal point



EXP(0)=1

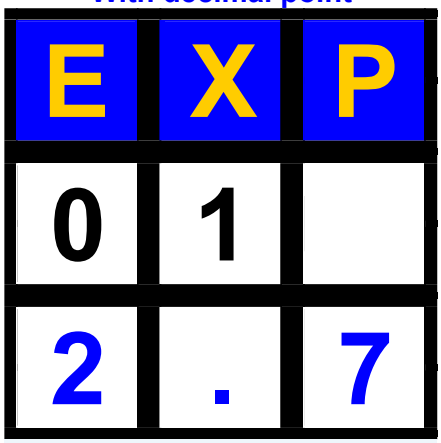
Exponential Function

Without decimal point



EXP(1)=2.7

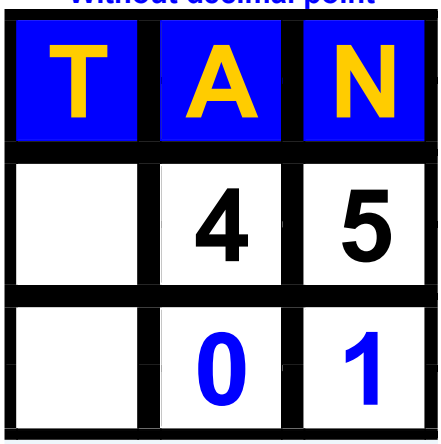
With decimal point



EXP(1)=2.7

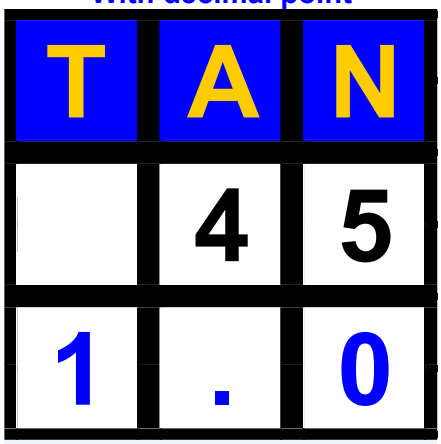
Tangent Function

Without decimal point



TAN(45)=1

With decimal point



TAN(45)=1

Sine Function

With decimal point

S	I	N
	3	0
	.	5

SIN(30)=.5

With decimal point

S	I	N
	4	5
	.	7

SIN(45)=.7

Cosine Function

With decimal point

C	O	S
	3	0
	.	9

COS(30)=.9

With decimal point

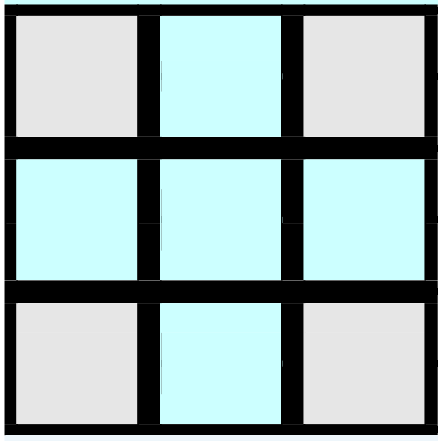
C	O	S
	4	5
	.	7

COS(45)=.7

Solving a Mathematical Functions Cube Step by Step

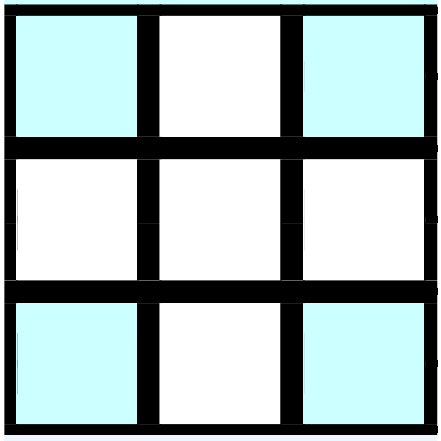
In this example, a step by step solving process is applied to the Mathematical Functions Cube, just described before. Note that we only need to solve a *single* Face out of six. We will solve a Face for EXP(1)=2.7.

Solve the Cross First



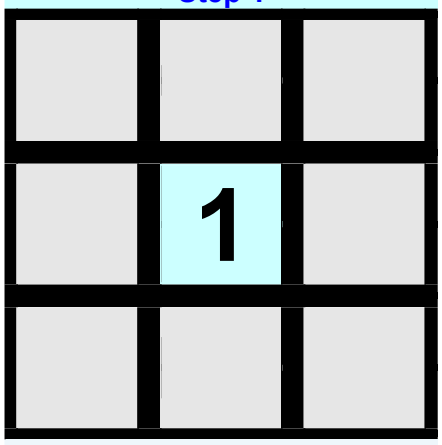
Standard Rubik's Cube Solving

Then Solve the Corner Cubes



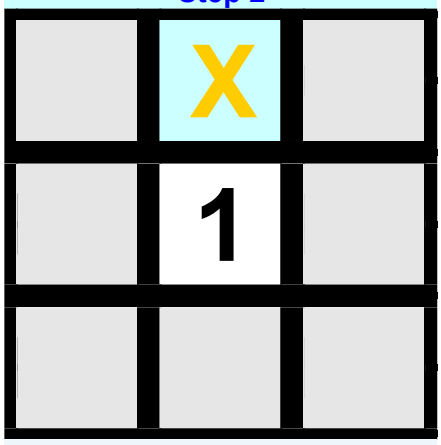
Standard Rubik's Cube Solving

Step 1



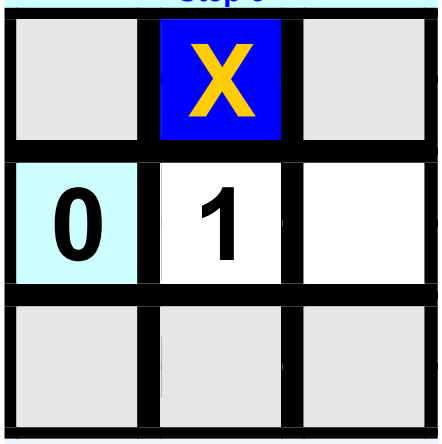
Center Cube **1** on Front Face

Step 2



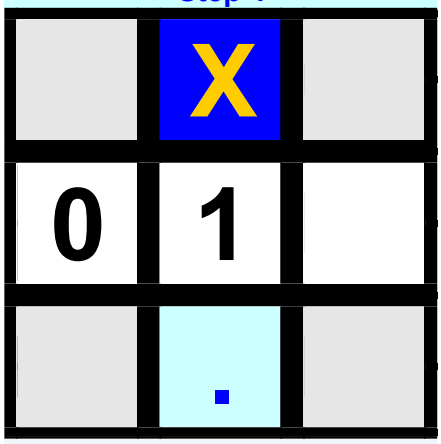
Top Layer: Edge Cube **X**

Step 3



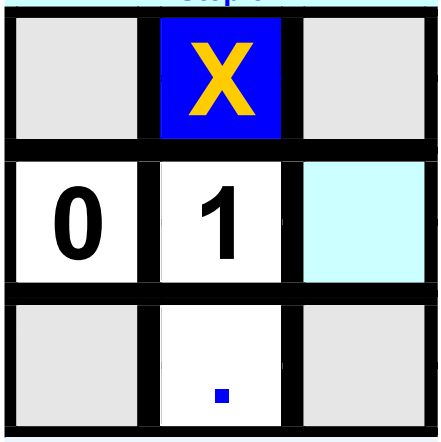
Middle Layer: Edge Cube **0**

Step 4



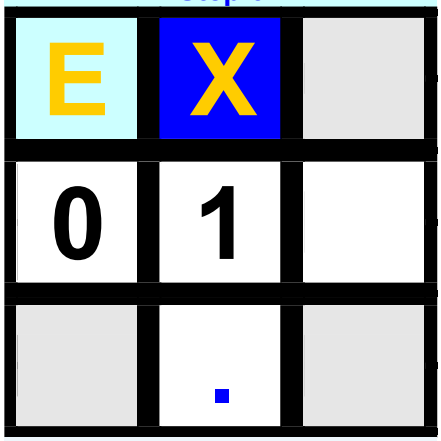
Bottom Layer: Edge Cube **.**

Step 5



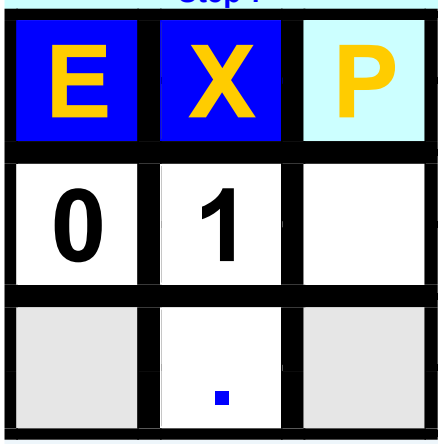
Middle Layer: Edge Cube **White**

Step 6



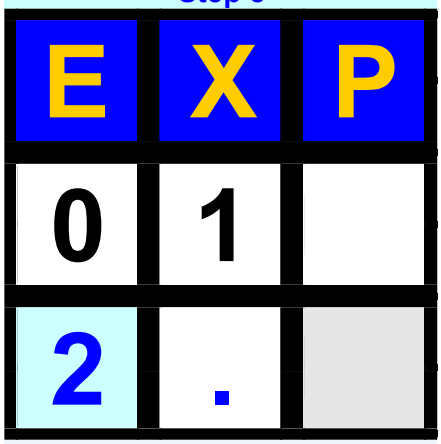
Top Layer: Corner Cube **E**

Step 7



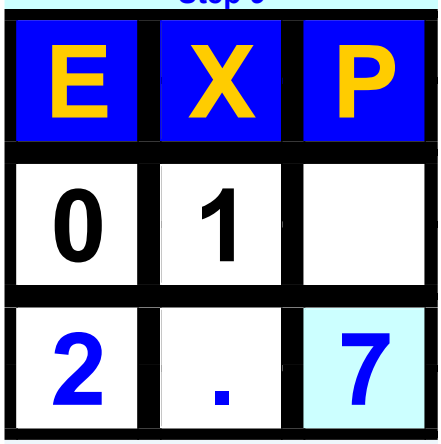
Top Layer: Corner Cube **P**

Step 8



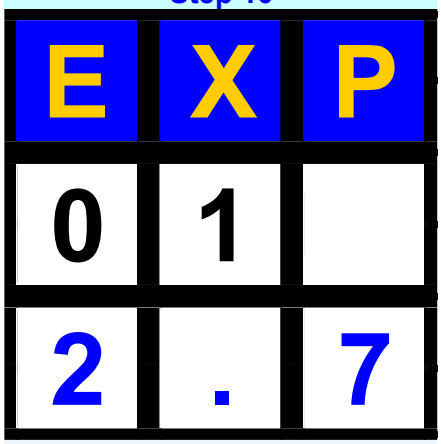
Bottom Layer: Corner Cube **2**

Step 9



Bottom Layer: Corner Cube **7**

Step 10



That's it !

Print out this page and fill in the blank faces with *your* data. Then try to design your *own* Cube.

TL	TC	TR
ML	MC	MR
BL	BC	BR

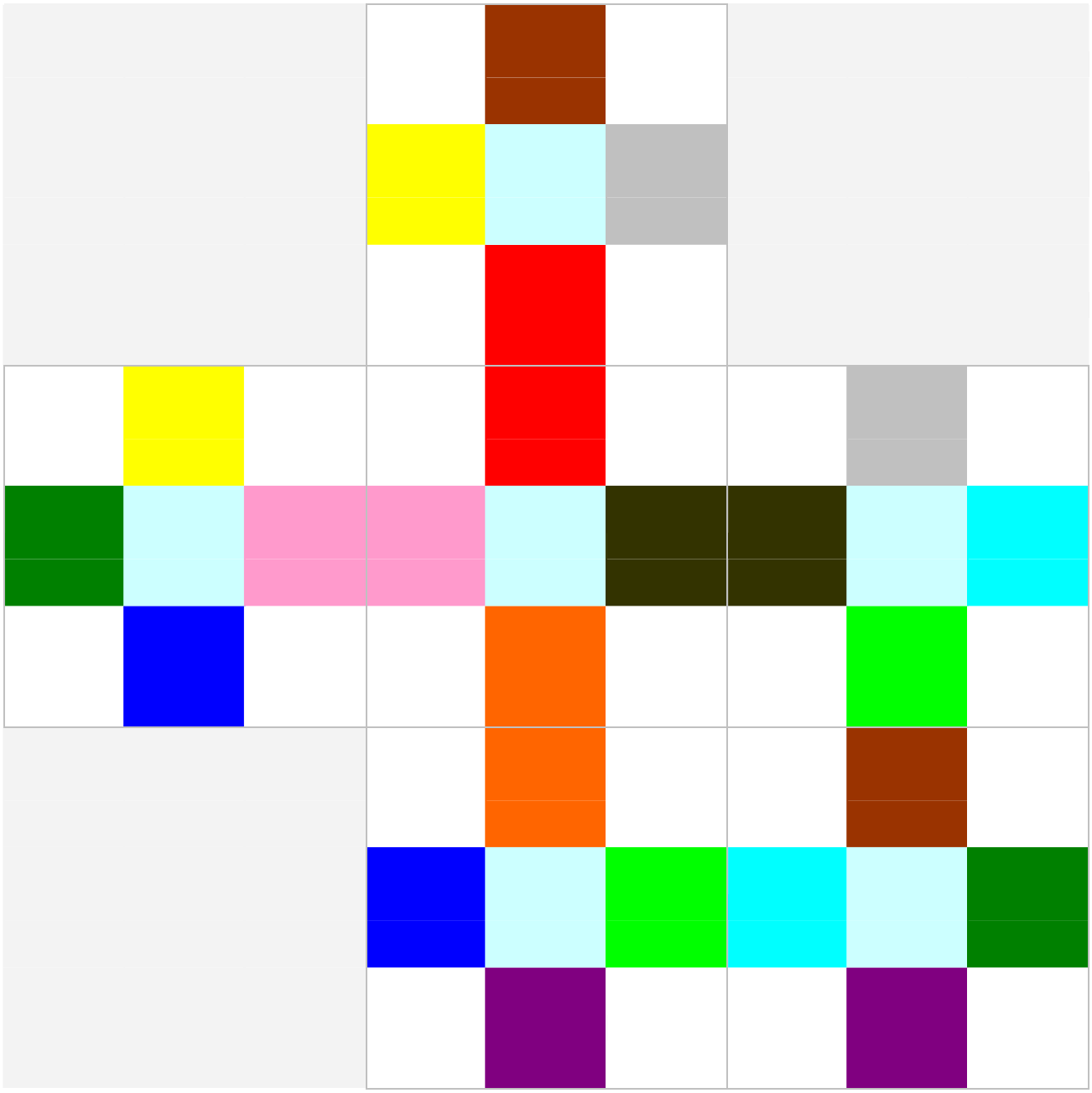
Corner Cubes Final Check

There are 8 Corner Cubes and 3 faces per Corner Cube. In the diagram below, each Corner Cube is displayed in 8 different colors and with the same color applied to each of its 3 faces. This diagram can be used as a convenient *visual aid* to check Design Rules (DRC).

	Up		
Left	**F**ront	**R**ight	**B**ack
	Down	**B**ack	

Edge Cubes Final Check

There are 12 Edge Cubes and 2 faces per Edge Cube. In the diagram below, each Edge Cube is displayed in 12 different colors and with the same color applied to each of its 2 faces. This diagram can be used as a convenient *visual aid* to check Design Rules (DRC).



	Up		
Left	**F**ront	**R**ight	**B**ack
	Down	**B**ack	

Texture Template

This is a texture template that can be printed out and used for writing down numbers and letters by hand *prior to* texture design. All is needed are pencil, rubber…and time.