Sundial Cube Design

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WebSites	http://www.mementoslangues.fr/	http://www.randelshofer.ch/

Introduction

A **Sundial Cube** is a 3x3x3 **Rubik's Cube** used to measure the time by the position of the Sun. This is a vertical sundial with time displayed on the cube front face. However, to be fully functional as a *real* sundial, the cube needs an accessory, which is a 'shadow maker' called a *gnomon*. When the cube is not used as a sundial, it can nevertheless be used as a Super Cube, which is a design variation of a Rubik's Cube with marker strokes drawn on some stickers to show the orientation of side parts. The Sundial Cube *noon lines* can then be used as Super Cube strokes.

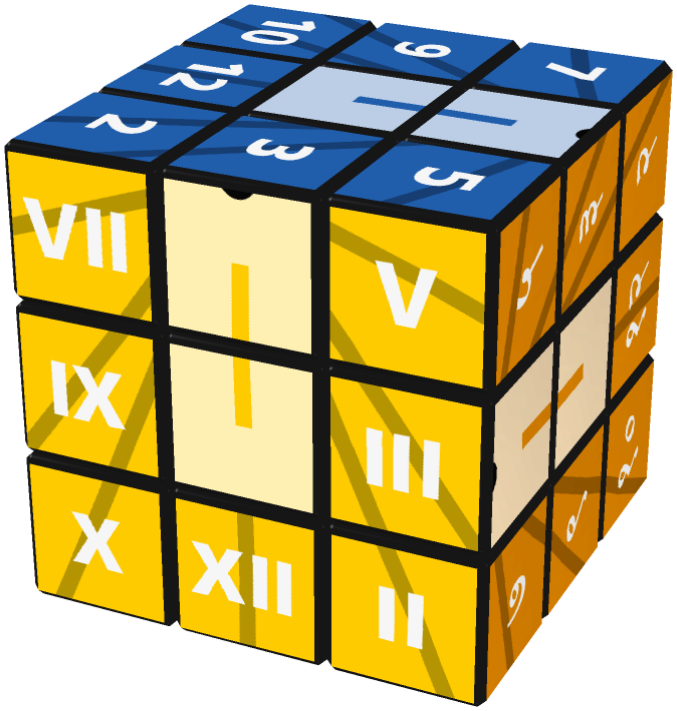
Sundials – Useful Links	
http://en.wikipedia.org/wiki/Sundial	http://en.wikipedia.org/wiki/List_of_cities_by_latitude

The vertical sundial cube can be read in six Capital cities, all around the Globe: London (Great Britain), Roma (Italy), Beijing (PRC), Seoul (South Korea), New Delhi (India) and Riyadh (Saudi Arabia). Six kinds of numerals are used on the cube: Arabic, Roman, Chinese/Japanese, Korean (Hangul), Hindi (Devanagari) and Arabic-Indic numerals. Moreover, the computed angle between a given hour-line and the noon hour-line is different for each Capital city, making this cube a precise solar clock.

There are **Virtual Cubes** that can be *virtually* rotated and twisted on a computer screen and **Real Cubes** that can only be *physically* rotated and twisted by hand. A **Texture** is laid down on a Virtual Cube whereas real **Stickers** are stuck down on a Real Cube. A Sundial Cube is designed by placing numerals and hour-lines on a texture which is then laid down on a Virtual Cube (see http://www.randelshofer.ch/ for more details). The time of the day can be displayed on a *selected* Cube face by rotating and twisting some parts of the Cube and by using a gnomon. When this has been achieved, we say that the Cube has been *solved*.

The following example shows the initial state of a *Virtual* Vertical Sundial Cube, where Roman numerals are displayed on the front face, Devanagari numerals on the right face and Arabic numerals on the top face. Noon lines can also be used as Super Cube strokes.

Vertical Sundial Cube

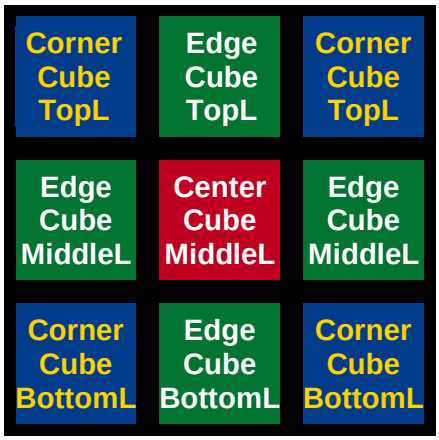


Vertical Sundial Cube Texture	Virtual Vertical Sundial Cube

Download **CubeTwister** from: http://www.randelshofer.ch/

Terminology

In a 3x3x3 **Rubik's Cube**, there are 8 *Corner Cubes*, 12 *Edge Cubes*, 6 *Center Cubes* and 6 *Cube Faces*. There are also 4 Corner Cube faces, 4 Edge Cube faces and 1 Center Cube face *per Cube Face*, as shown below.



There are 1 face per Center Cube, 2 faces per Edge Cube and 3 faces per Corner Cube.

There are also 3 horizontal *Layers* called *Top*, *Middle* and *Bottom Layers*.

Cube Lexicon		
English	**Français**	**Deutsch**
Cube	Cube	Würfel
cubie, cube	cube, petit cube	Würfelteil, Teil des Würfels
face	face	Seite, Seitenfläche
front face	face avant	vordere Seite, vorne
back face	face arrière	hintere Seite, hinten
left face	face gauche	linke Seite, links
right face	face droite	rechte Seite, rechts
top face	face supérieure	obere Seite, oben
bottom face	face inférieure	untere Seite, unten
sticker	étiquette (autocollante), plaquette	Kleber, Farbkleber
tile	tuile, plaquette	Plättchen, Farbplättchen
center cube, center	cube central, centre	Mittelwürfel, Mittelstein, Mitte
edge cube, edge	cube-arête, arête	Kantenwürfel, Kantenstein, Kante
corner cube, corner	cube de coin, coin	Eckwürfel, Eckstein, Ecke
layer	couronne	Schicht, Scheibe
top layer	couronne supérieure	obere Schicht, obere Scheibe
middle layer	couronne intermédiaire	mittlere Schicht, mittlere Scheibe, Mittelschicht, Mittelscheibe
bottom layer	couronne inférieure	untere Schicht, untere Scheibe
orientation, direction	orientation	Orientierung
to solve	résoudre	lösen, zusammen drehen
to twist	pivoter	drehen
to rotate	tourner, effectuer une rotation	drehen
clockwise	dans le sens horaire	im Uhrzeigersinn
anticlockwise, counter-clockwise	dans le sens anti-horaire	im Gegenuhrzeigersinn

Vertical Sundials

The 'shadow-maker' of a sundial is called a *gnomon*. The linear feature that casts the shadow from which the time can be read is often called a *style* (a thin rod or a sharp, straight edge). The style should be parallel to the Earth's axis of rotation, and point to the celestial pole. The sun casts a shadow from the gnomon or style to a surface called the *dial face* or *dial plate* (often shortened to *face*).

In common sundial designs, the sun casts a shadow from its style onto a flat surface marked with lines indicating the hours of the day. As the sun moves across the sky, the shadow-edge progressively aligns with different hour-lines on the plate. Such designs rely on the style being aligned with the axis of the Earth's rotation.

In the common *vertical* dial, the shadow-receiving plane is aligned vertically. The style's angle with vertical (φ) must equal the co-latitude, which is 90° *minus* the sundial's geographical latitude (λ):

$$\varphi = 90° - \lambda$$

However, the line of shadow does not move uniformly on the face: the sundial is not *equiangular*. If the face of the vertical dial points directly South, the angle of the hour-lines is instead described by the formula:

$$\tan(\theta) = \mathbf{cos}(\lambda)\times\tan(15°\times t)$$

where λ is the sundial's geographical latitude, θ is the angle between a given hour-line and the noon hour-line (which always points due North) on the plane, and t is the number of hours before or after noon. For example, the angle θ of the 3 pm hour-line would equal the arctangent of cos(λ), since tan(45°) = 1. Interestingly, the shadow moves *counter-clockwise* on a South-facing vertical dial, whereas it runs *clockwise* on horizontal and equatorial dials.

And finally, there are adjustments to calculate *local* clock time from a sundial reading:

1- Summer (daylight saving) time correction
2- Time zone (Longitude) correction
3- Equation of time correction

For more on these corrections, see http://en.wikipedia.org/wiki/Sundial.

Vertical Sundial Angles – tan(θ) = cos(λ)×tan(15°×t)									
Latitude λ	**Longitude**	**Capital City**	**Country**	**θ (t=1h)**	**θ (t=2h)**	**θ (t=3h)**	**θ (t=4h)**	**θ (t=5h)**	**φ**
51°30'N	0°08'W	London	Great Britain	9.5°	19.8°	31.9°	47.2°	66.7°	38.5°
41°54'N	12°29'E	Roma	Italy	11.3°	23.3°	36.7°	52.2°	70.2°	48.0°
39°54'N	116°23'E	Beijing	PRC	11.6°	23.9°	37.5°	53.0°	70.7°	50.0°
37°33'N	126°59'E	Seoul	South Korea	12°	24.6°	38.4°	53.9°	71.3°	52.5°
28°38'N	77°13'E	New Delhi	India	13.2°	26.9°	41.3°	56.7°	73.0°	61.3°
24°43'N	46°43'E	Riyadh	Saudi Arabia	13.7°	27.7°	42.3°	57.6°	73.6°	65.3°

Sundial Numerals												
Arabic	1	2	3	4	5	6	7	8	9	10	11	12
Roman	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Chinese/Japanese	一	二	三	四	五	六	七	八	九	十	十一	十二
Korean (Hangul)*	하나	둘	셋	넷	다섯	여섯	일곱	여덟	아홉	열	열하나	열둘
Hindi (Devanagari)	१	२	३	४	५	६	७	८	९	१०	११	१२
Arabic-Indic	١	٢	٣	٤	٥	٦	٧	٨	٩	١٠	١١	١٢

* The Korean language has two regularly used numeral systems, native Korean and Sino-Korean. Native Korean numerals are used for hours from 0:00 to 12:00 whereas Sino-Korean numerals are used for hours from 13:00 to 24:00. For example, *se si* (세 시) means '03:00' or '3:00 a.m./p.m.' and *sip-chil si* (십칠 시) means '17:00'. Native Korean numerals are therefore used for displaying hours on sundials in the 12-hour system.

Horizontal Sundials

In the common *horizontal* sundial, the shadow-receiving plane is aligned horizontally. The style's angle with the plane must equal the sundial's geographical latitude (λ).

As is also the case for the vertical sundial, the line of shadow does not move uniformly on the plane: the sundial is not *equiangular*. If the style points directly North, the angle of the hour-lines is instead described by the formula:

$$\tan(\theta) = \mathbf{sin}(\lambda)\times\tan(15°\times t)$$

where λ is the sundial's geographical latitude, θ is the angle between a given hour-line and the noon hour-line (which always points due North) on the plane, and t is the number of hours before or after noon. For example, the angle θ of the 3 pm hour-line would equal the arctangent of sin(λ), since tan(45°) = 1. Interestingly, the shadow moves *clockwise* on horizontal and equatorial dials whereas it runs *counter-clockwise* on a South-facing vertical dial.

Note that the angle θ between a given hour-line and the noon hour-line is not the same for the vertical and horizontal sundials.

Horizontal Sundial Angles – tan(θ) = sin(λ)×tan(15°×t)									
Latitude λ	**Longitude**	**Capital City**	**Country**	**θ (t=1h)**	**θ (t=2h)**	**θ (t=3h)**	**θ (t=4h)**	**θ (t=5h)**	**λ**
51°30'N	0°08'W	London	Great Britain	11.8°	24.3°	38.1°	53.6°	71.1°	51.5°
41°54'N	12°29'E	Roma	Italy	10.2°	21.1°	33.8°	49.2°	68.2°	42.0°
39°54'N	116°23'E	Beijing	PRC	9.8°	20.4°	32.8°	48.1°	67.4°	40.0°
37°33'N	126°59'E	Seoul	South Korea	9.3°	19.4°	31.3°	46.5°	66.2°	37.5°
28°38'N	77°13'E	New Delhi	India	7.3°	15.5°	25.7°	39.8°	60.8°	28.7°
24°43'N	46°43'E	Riyadh	Saudi Arabia	6.4°	13.6°	22.7°	35.9°	57.3°	24.7°

Combined Sundials

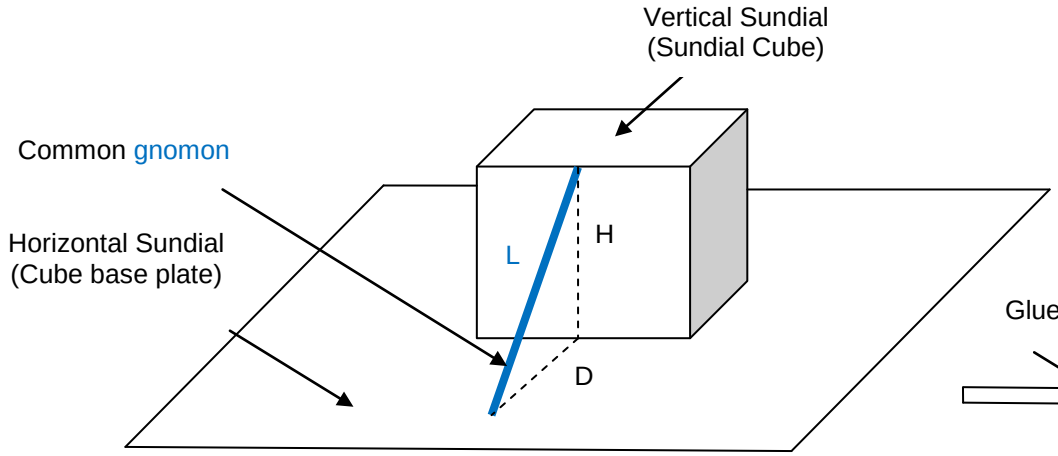
A vertical and an horizontal sundial can be combined by sharing the *same* gnomon, because their gnomon angles are *complementary* i.e. they sum up to 90°.The horizontal sundial plane can also be used as a base plate for the cube. In that case, the common gnomon should only be fastened onto the plane and not onto the cube. The common gnomon length *L* and the distance of the gnomon far end from the cube *D* are given by:

$$L = H/\sin(\lambda) \text{ and } D = H/\tan(\lambda)$$

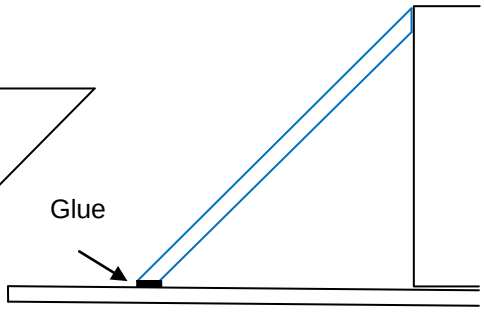
where λ is the sundial's geographical latitude and *H* is the cube height, usually equal to 57 mm for a regular Rubik's cube.

Combined Sundials – Common Gnomon Length						
Latitude λ	**Longitude**	**Capital City**	**Country**	**H mm**	**D mm**	**L mm**
51°30'N	0°08'W	London	Great Britain	57	45	73
41°54'N	12°29'E	Roma	Italy	57	64	85
39°54'N	116°23'E	Beijing	PRC	57	68	89
37°33'N	126°59'E	Seoul	South Korea	57	74	94
28°38'N	77°13'E	New Delhi	India	57	104	119
24°43'N	46°43'E	Riyadh	Saudi Arabia	57	124	136

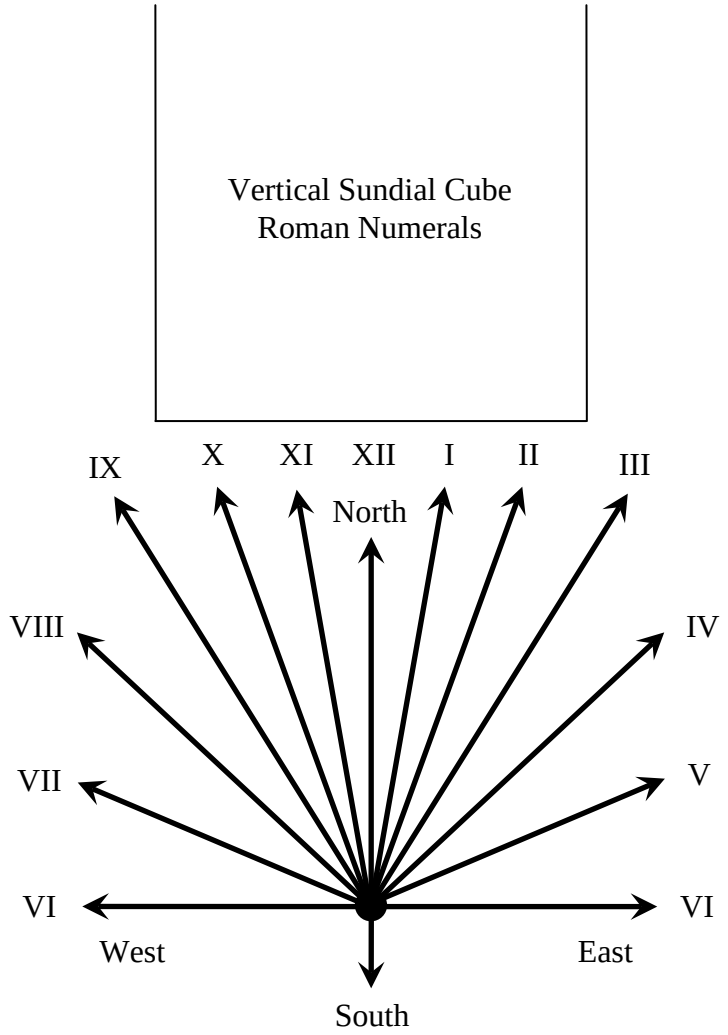
The gnomon can be made from a 3 mm diameter plastic rod, which should be bevelled at both ends. The rod lower end can then be glued onto the base plate whereas the upper end simply leans against the top center cubie.



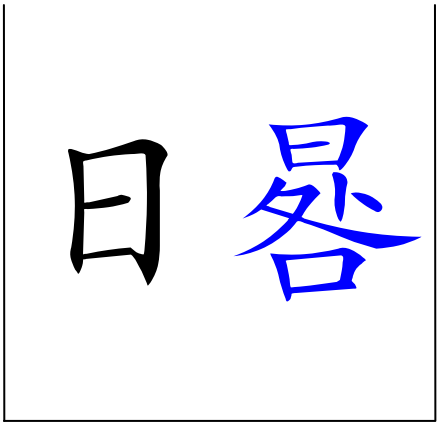
Combined sundials schematic

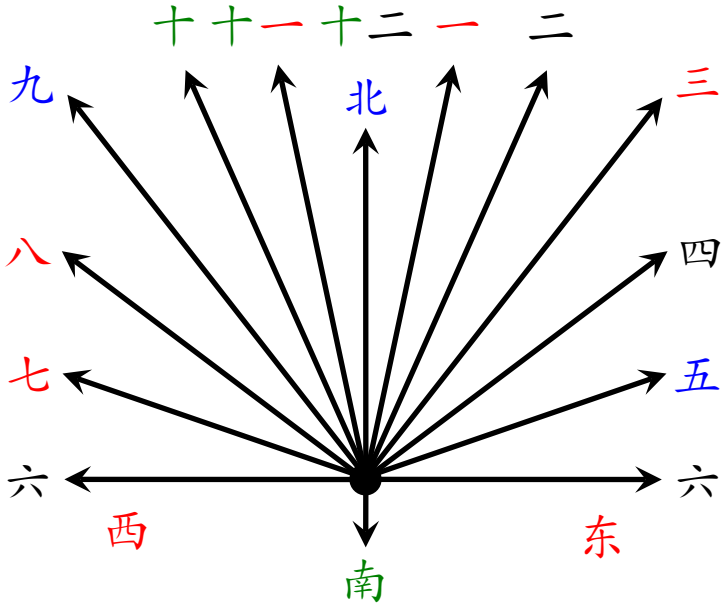


Common gnomon shape (bevelled rod)

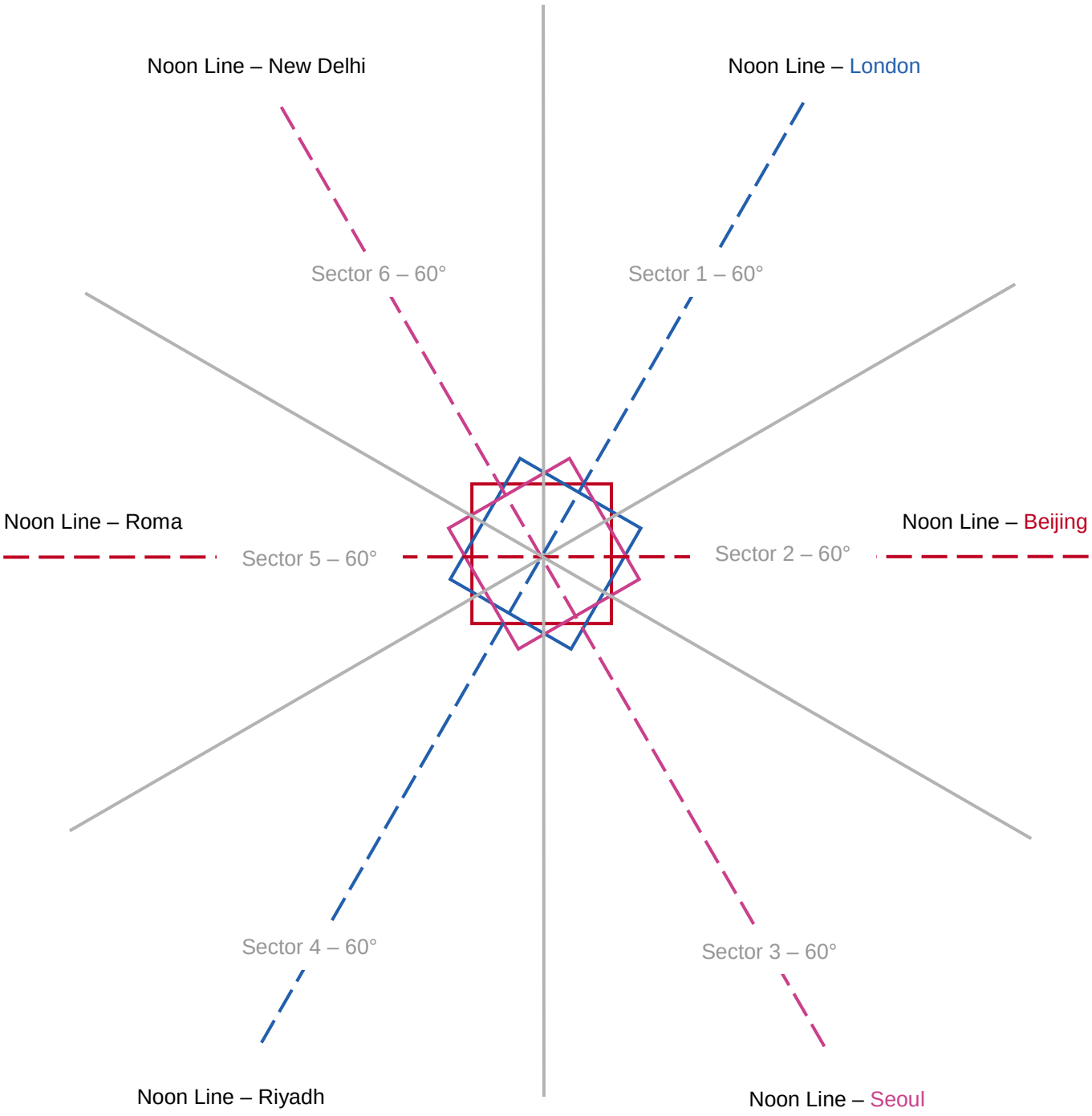


Combined sundials layout
Roman numerals





Combined sundials layout
Chinese numerals



Schematic of 6 combined sundials sharing the *same* base plate – Sundial Cube stands on center

Cardinal Directions

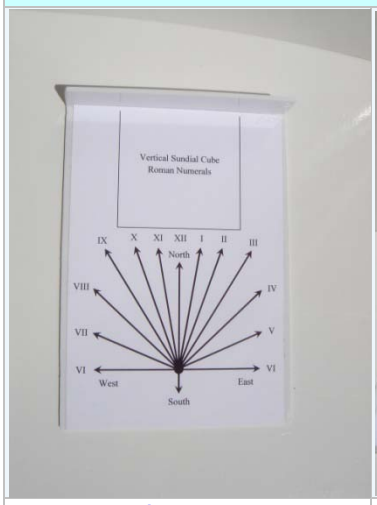
Cardinal directions can be obtained from a sundial, as follows:

- By observing the sun in the sky, wait until it is at its maximal height.
- Then turn the sundial until the gnomon shadow edge aligns with the noon line.
- When this has been achieved, the gnomon is pointing due South, so North is opposite, East is on the right and West is on the left.

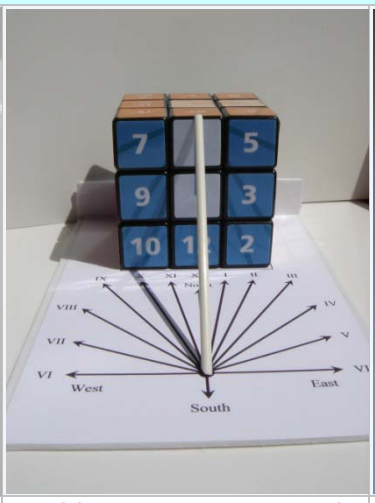
At another time of the day, the South direction can also be found if one knows both local time and the 3 time adjustments. Proceed as follows:

- Calculate the solar hour from the local time and time adjustments.
- Then turn the sundial until the gnomon shadow edge aligns with the solar hour line.
- When this has been achieved, the gnomon is pointing due South, so North is opposite, East is on the right and West is on the left.

Sundial Cube Photos

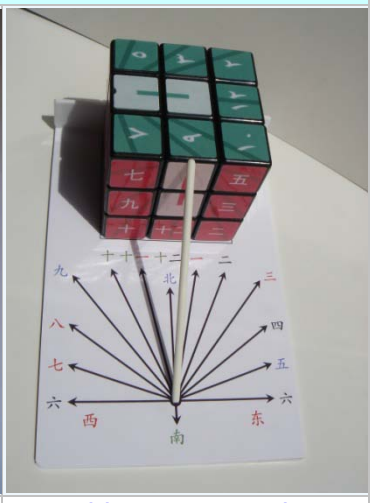


Base plate – Roman



Arabic & Roman numerals

Base plate – Chinese



Chinese numerals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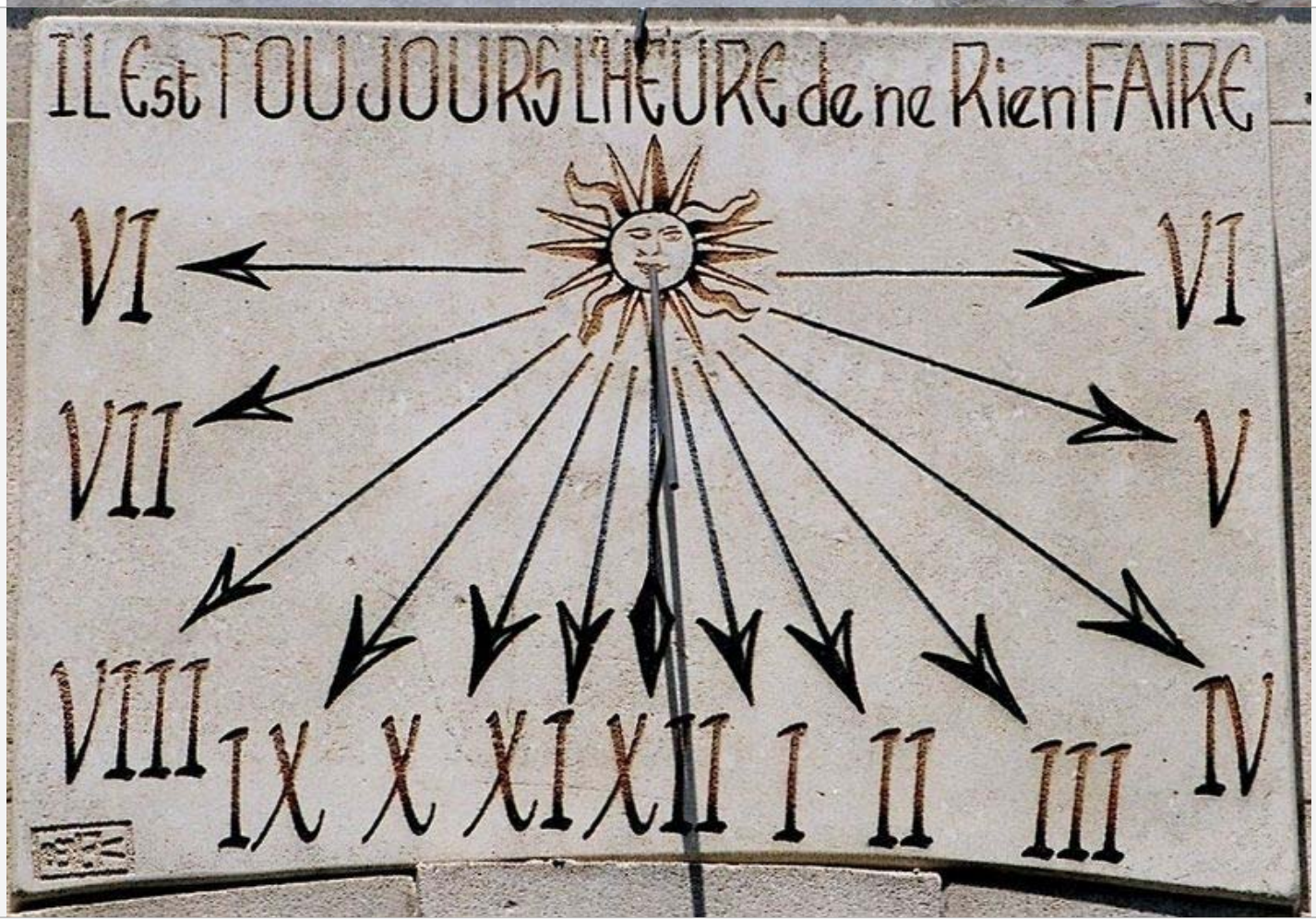
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