from: *Music and Deep Memory: Speculations in ancient mathematics, tuning, and tradition, in memoriam Ernest G. McClain.*

97-98

THE GEODETIC AND MUSICOLOGICAL SIGNIFICANCE OF THE SHORTER SIDE LENGTH OF THE PARTHENON AS HEKATOMPEDON OR 'HUNDRED-FOOTER'

Richard HEATH

This note responds to Kapraff and McClain's preceding paper, in which they discover a many-faceted musical symbolism in the Parthenon. Specifically, Ernst Berger's new measurements include the shorter side of the triple pedestal of the monument as an accurate length to represent one second of the double meridian of the earth. By applying a knowledge of ancient metrology, Anne Bulckens' doctoral derivations of a root foot can resolve to a *pygme* of 9/8 feet, of which one second of latitude would contain 90 such feet. However, as a '*hundred footer*', the foot length should then be 81/80 (1.0125) feet, the ratio of the syntonic comma. This would indicate a replacement, by Classical times, of the geographical constant of 1.01376 feet within the model of the earth since the original model, by the late megalithic, assumed that the meridian was exactly half of the mean circumference of the earth. These alternative geographical constants co-incidentally represent the ubiquitous theme in ancient musicology of the transition between Pythagorean and just tunings and their respective commas.

It has been thought that a term used in Archaic Greek to define the size of a sacred building, the '*hundred footer*' or *hekatompedon*, could refer to its area being equal to 10,000 square feet, which in a square building would be 100 feet on each side. The Parthenon, the sides of which are in the ratio of 9:4, has an area 36 so that as a square it would be six on a side and, if these were 100 cubits of 3/2 feet, then the four sides of the Parthenon would be 100 feet wide.

Instead, recent measurements in 1982 by Ernst Berger[1] found that the width of the top surface of the *stylobyte* was just over 101.25 feet and that the most frequently occurring length was 857.6 mm. Anne Bulckens'[2] corresponding foot measure for this would be a step of 2.5 feet, each of 9/8 (1.0125) feet (to one part in 2500). This unit is well within the range of foot lengths within historical metrology, and is called a *pygme* after the size of small men, first mentioned when *Herakles* was travelling back from India[3]. The four sides of the Parthenon would then be 90 such feet across.

However, should the four sides be divided by 100, the required foot length of 101.25 feet, divided by one hundred, becomes a microvariation of the English foot, namely 81/80 (1.0125) feet; a length which is identical with the syntonic comma which is itself a ratio crucial to the history of ancient tuning theory, as the ratio found between them are pure Pythagorean tones and their counterparts within just tuning, in which string lengths are given specific whole number lengths to specify their pitches, intellectually.

A recent article by Jay Kapraff and Ernest McClain[4] observes that the width of the Parthenon symbolically defined one second of latitude (taking surface lengths as linear fractions of latitude) if a double meridian length had been estimated to be

1 Berger, E., ed. (1986) *Parthenon-Kongress Basel*, 2 Vols, Mainz: Philipp von Zabern.

2 Bulckens, A.M. (1999) The Parthenon's Main Design Proportion and its Meaning, [Ph.D. Dissertation], Geelong: Deakin University, 269 pp. ; (2001) The Parthenon's Symmetry in Symmetry: *Art and Science* (*Fifth Interdisciplinary Symmetry Congress and Exhibition of the ISIS-Symmetry*), (Sydney, 2001), no. 1-2, pp. 38-41.

3 Philostrates of Lemnos (c. 190 – c. 230 AD) Imagines Heracles among the Pygmies, see Loeb Classical Library

4 The Proportional System of the Parthenon, in preparation for the *In Memoriam* volume for Ernest McClain (1918-2014)

close to its modern estimation (being then within 0.003%). This implies that geodetic design was intended for the four sides of the Parthenon, as smaller than the north south circumference of the earth by one 3,600th of one 360th part. This means the implied geodetic metrology of the Parthenon can be compared with monuments built two thousand years earlier, such as Stonehenge and the Great Pyramid of Giza, within which the relationship of the mean earth was specified relative to the polar radius, using the same metrological system.

In the ancient model of the earth, recovered[5] by John Neal[6] and John Michell[7], three different approximations of π were used to allow for the distortion of the rotating planet over what its mean, or perfectly spherical, circumference would be. In that model, the circumference of the mean earth was taken as being identical to the actual, double-meridian length, both lengths then assumed to be 44 times 12^6 (131,383.296) feet or 24,883.2 miles. Were the Parthenon to have been built using this model then its four sides would be 101.376 feet in length and one hundredth of this would be a foot of 1.01376 feet, a foot known as the '*Standard Geographical Greek foot*[8]'. (This fraction of 1.01376 (3168/3125) is called the geographical constant because it relates to variation of north-south degree lengths of latitude due to the shape of the earth[9], and does not define the meridian as equalling half the length of the mean earth's circumference.)

The double meridian in the model (equal to the mean circumference of the earth) was 24,883.2 miles long while a modern estimate for the double Meridian length is 24,859.868 miles and this is within one part in 3,200 of the ratio between the feet 1.01376 (3168/3125) and 1.0125, the 81/80 foot that would make the Parthenon a 'hundred footer'. It is therefore reasonable to assume that, between the building of Stonehenge and Great Pyramid (by 2,500 B.C.) and the building of the Parthenon (designed by 447 B.C.), an actual length for the Meridian had been measured and that this had superseded the assumption, within the old model, that the meridian was half the length of the mean earth circumference.

Further to this, one can see how the transition from Pythagorean to just tuning systems[10] is strangely present in the relationship between the mean earth circumference and the actual meridian length, since the geographical constant of 1.01376 is near identical to the Pythagorean comma of 1.0136433 while the (chosen) ratio 1.0125 is the syntonic comma and this, times 100, is near identical to the actual length of one second of latitude which would be 100 times 1.0128 feet[11], just one third of an inch different from a more modern exactitude.

The Parthenon 'Hundred footer' was able to dimensionally reference one second of the Meridian by having its shorter sides one hundred feet of 1.0125 feet long. Aligned to north, this presented accurate Classical knowledge of the Meridian's length. The monument expresses other musicological features via its metrology: the 81/80 foot unit is 125/128 of the Athenian foot of 1.0368 feet, a musical interval called the minor diesis, also found within just intonation and equaling the deficiency of three major thirds to the octave.

5 Michell by 1980 and Neal, fully formed, by 2000.

6 Neal, John (2000) All Done With Mirrors, *Secret Academy*, London.

7 Michell, John (1982) Ancient Metrology, Pentacle Books, Bristol, 1982; (2008 new ed.) Dimensions of Paradise, Inner Traditions: Rochester.

8 Using the terminology developed by John Michel and John Neal.

9 A day in angle on the Equator was defined as being 360,000 feet whilst the north-south mean degree (at 51 degrees) was 360,000 longer feet, namely of Greek geographical feet of 1.01376 feet.

10 The latter prevalent in other aspects of the monument, see Kappraff, J. and McClain, E.G. (2005: Spring–Fall) The Proportions of the Parthenon: A work of musically inspired architecture, *Music in Art: International Journal for Music Iconography,* Vol. 30/1–2.

11 A non-harmonic 79/78 feet.