1 Astronomical Musicality within Mythic Narratives

by Richard Heath December 2013

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3 I open this paper with a diagram of its structure (figure 1). This purportedly ancient symbol came to

4 light around 1914 as the (nine fold) *Enneagram* [BLAKE,1996]: Literary analysis has shown ancient writers

5 using very similar **Ring Compositions** [DOUGLAS, 2010] as just one of many techniques, often employed

6 within mythic texts to insert hidden **structures of meaning**.



8 Figure 1 An Enneagram used as ring composition for this paper to help author and reader understand its narrative
 9 structure.

10 I will be introducing another such technique rediscovered by Ernest McClain [MCCLAIN, 1976, 1978] 11 in which he has "offered a persuasive explanation of crucial passages in texts of world literature—the Bible, the 12 Rig Veda, the Egyptian Book of the Dead, and Plato's dialogues -- that have defied experts in the concerned 13 disciplines." - Wikipedia. His work finds harmonic parallelism within texts and finds a purpose for the anomalous often "astronomical" numbers, found within mythic narratives whose meaning is explainable by 14 15 reconstructing a unique array (a matrix) of whole numbers lesser than the number within the text, numbers 16 evoked purely by consequence of **limiting** the harmonic generative power to that number. McClain finds that a 17 limit's matrix, naturally shaped like a mountain, explains the plot elements, events and characters of the 18 narrative, which run *parallel* to the harmonic concerns found within these mountains. The most significant 19 question emerging from such harmonic parallelism within texts is why the harmonic facts implicit within

- 20 different limiting numbers were ever thought an important substratum for mythic narrative. What could have
- 21 made harmonic facts relevant to spiritual tales involving divine worlds and heroic dramas?

22 Part One: Possible Sources of Astronomical Musicality

23 Evidence of techniques like ring composition and harmonic parallelism within the narratives of the 24 ancient world throw light on the intellectual world of their authors. Ancient musical knowledge must have come 25 to Just tuning long before Greek music, quite possibly because of two sources of musical information not usually 26 associated with early musical *tuning by ear*: firstly, the **early number field** is the original template upon which 27 musical harmony is based [see pages 4-5]; and secondly, the prehistoric geocentric astronomy which preceded the ancient world could compare counted astronomical time-periods, and therefore discovered the rational 28 29 wholetone and semitone intervals between the lunar year, Jupiter and Saturn [HEATH, 2004, 2007]. This would 30 mean that the cultures preceding the ancient Near East, though lacking an arithmetic, had inherited a metrology 31 which could count numbers as collections of unit lengths and was also able to calibrate geometrical structures 32 with their metrology, as is found within the large built structures of Egypt and Mesopotamia.



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Figure 2 The wholetone and semitone interval between the lunar year and synods of Jupiter and Saturn presented
within a right angled triangle as if three musical strings.

- In *Matrix of Creation* [HEATH 2002] I reported astronomical intervals, findable by the megalithic
 methods of north west France (see below). These express *exact musical ratios* relative to the lunar year, Jupiter
- a Pythagorean wholetone (8:9) and Saturn a Just semitone $(15:16)^1$. These two intervals are only found together
- 39 in Just tuning. My recent collaboration with musicologist Ernest McClain led to my developing a web application

¹ Note that Hesiod's Cosmogony has Saturn as Cronos, being deposed by Jupiter as Zeus and relegated to a "small island", parallel perhaps to his Just semitone status as opposed to Jupiter's Pythagorean wholetone with respect to the Moon, which is then a dominating control over the Moon by Jupiter, the nearest and largest gas giant.

40 called *Harmonic Explorer*² which enables rapid calculation of and navigation between the harmonic worlds 41 generated by limiting numbers. Through this I discovered that the musical ratios of Jupiter and Saturn require, 42 at least, the Just tuning matrix for the limiting number of 1440 units. Driving this matrix is the synodic comma 43 [ratio 80:81] uniquely arising between the Pythagorean and Just tone sets of Just intonation, but in this case between the Moon's synodic month and the Earth's rotation. If realised, deep links could have been forged in 44 the late prehistoric between musicology, astronomy and myth, Later, traditional symbols and narratives could 45 have been crafted to preserve the astronomical and musical realities within creative textual dramas, about 46 47 planetary gods.

In 2010, my brother and I [HEATH and HEATH, 2010] surveyed a megalithic monument of the fifth 48 49 millennium BCE which counted the days of at least two different types of year, through accumulating a constant 50 unit of length to form what is then a **numerical length**. These counts were then juxtaposed (within a monument called the Le Manio Quadrilateral) for comparison of lengths using a right angled triangle. One notes this as 51 52 being exactly the same principle one might use when comparing two string lengths, where a series of intervals held between string lengths can create a melodic scale, string ratios containing only the three superparticular 53 ratios of Just tuning, two wholetones [8:9 and 9:10] plus a semitone [15:16]. This illustrates the means whereby 54 55 the harmonic relationships between the Moon, Jupiter and Saturn *could* have come to be associated with string lengths and how the celestial regions might originally have come to be seen as a "harmony of the spheres", these 56 anciently thought of as titans and gods, and only then as a fitting *implicit* subject for the mythic narratives of the 57 ancient world. 58



Figure 3 The three solar year day-inch count to be found within Le Manio's Quadrilateral, expressed as the
invariant right triangle for the interval between the solar and lunar years.

² Available at http://harmonicexplorer.org

62 The matrices of Ernest McClain could also have developed within the megalithic alongside a pre-63 arithmetic astronomy using right triangles, which can use the all-important units held as the *differences* between 64 two counted lengths of time. It is this difference, dividing into two string lengths, which reveals a superparticular ratio uniquely expressive of a given interval which, when harmonic, is a musical interval. We indeed see in the 65 Le Manio Quadrilateral a formation of what was probably the original megalithic vard, being such a differential 66 67 unit held between day counts for three lunar years and three solar years, a length of 32 and 5/8th day-inches³. This simple way of handling numbers and ratios is quite unfamiliar today and this has made what the megalithic 68 69 were achieving within their monuments obscure. Instead, a common explanation given for these monuments is 70 as providing settings for unspecified sacred rituals, just as a modern religious building might be used. If, 71 however, planets were being equated in the megalithic with gods and if their planetary time periods exhibited 72 harmonic intervals between one another, then some of these monuments are better explained as technical 73 structures whose aim became religious only when characterising these planets as important aspects of reality.

74 Another natural source of harmonic order can be found in the simplest intervals, which exist between 75 the smallest numbers themselves when cumulatively quantified, as the differential ratios containing their 76 products. Quite simply, all of what western music associates with harmony is generated by the differential "size" 77 of the first three prime numbers; two, three and five. Any number with only these numbers as its factors is, so to 78 speak, a **harmonic number** since it is capable of forming a harmonic interval with any other harmonic number. 79 There is a Nippur List of numbers⁴ showing only such harmonic numbers between 1 and 81, indicating how the early numbers were significant by 2,200 BC for Mesopotamian harmonic thought. The non-harmonic gaps 80 81 between the harmonic numbers separate the latter when viewed as "lengths", to form harmonic intervals between 82 and the gaps can only be made up of only two species (a) the other harmonic numbers and (b) the non-harmonic 83 numbers (which contain *any* factor other than 2, 3 and 5).

One needs to see each successive number as having length equal to one unit. The first six numbers 1-6 define the unitary ratios 1:2:3:4:5:6, sometimes called the *Senarius* or six unit metric, which expresses the octave 1:2, the fifth 2:3, the fourth 3:4, the major third 4:5 and the minor third 5:6. The first non-harmonic number, 7, causes a difference of two between 6:8, which is nothing new being a doubling of the fourth. But the next numbers give the two types of whole tone necessary to Just tuning. The next new species is 15:16, the Just semitone.

³ The Le Manio triangle, divided by its differential length, would have yielded the superparticular ratio of [32.6:33.6].

⁴ "Numerous examples exist of mathematical cuneiform tablets from the scribal schools of nineteenth and eighteenth century Larsa, Ur and Nippur, which contain thirty standard pairs of numbers with their reciprocals, encompassing all the sexagesimally regular numbers from 2-81. It was the musicologist, Ernest McClain21 who first suggested that these numbers, all in the form 2^p3^q5^r seem 'perfectly engineered to fit the specific needs of mathematical harmonics'". *NEW LIGHT ON THE BABYLONIAN TONAL SYSTEM* by Leon Crickmore, ICONEA2008

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Figure 4 The early number field is dominated by the first three primes, {2,3,5}, which generate musical octaves,
fifths, fourths, major and minor thirds through their successiveness [1:2:3:4:5:6] halted by seven, considered less
harmonic. This has a mythic interpretation of being the six days of creation after which God rests, opening the
Bible. Two wholetones [8:9:10] and semitone [15:16] completes the components of Just tuning without any resort to
a Pythagorean tuning procedure such as the Cycle of Fifths.

Each of the ratios within the ranges 1:2:3:4:5:6, 8:9:10, and 15:16, will be encountered later in the higher
number field, when inevitably doubled, tripled, quadrupled and so on. Something seems extraordinary in how
these multiples of the harmonic ratios then successively express the familiar order of the modes used within Just
tuning, starting with the unit length 24 [see figure below]. The first sequence is 24:27:30:32:36:40:45:48
describing the mode called Ionian by the Greeks, which starts with the interval 8:9 as 24:27, that is times three
(x 3). Then 9:10 is available as 27:30 (x 3), 15:16 as 30:32 (x 2), 8:9 as 32:36 (x 4), 9:10 as 36:40 (x 4), 8:9 as
40:45 (x5), as can be seen below.

103 This continues in a seamless way until the interval 9:10 as 72:80 (x8) which concludes the "last" mode, 104 called Aolian by the Greeks. This is possible because all of the six modal sequences effectively overlap, so that 105 Dorian starts with 9:10 as 27:30 (x 3), Phrygian starts with 15:16 as 30:32 (x 2), Lydian with 8:9 as 32:36 (x 4), 106 Mixolydian with 9:10 as 36:40 (x 4) and Aolian with 8:9 as 40:45 (x 5). This demonstrates an apparent 107 ascendancy of musical harmony (based upon the numbers 2, 3 and 5) in the way harmonic numbers are *perfectly* 108 packed or structured within the early number field. This extraordinary sequence of modes concludes with the distinctive comma of the Just tuning system, the syntonic comma of 80:81, which separates the Pythagorean 109 from the equivalent Just tone sets, just as the Pythagorean wholetone interval of 8:9 is 81 units long to the Just 110 111 wholetones 9:10 interval "length" of 80 units. Similarly, the Pythagorean Leimma (or heptatonic semitone) of 112 243:256 is one syntonic comma of 80:81 less in length than the Just semitone of 15:16.

Figure 5 How the numbers between 20 and 81 create enlarged whole and semi tones to, from 24 to 80, sequence the
six modes: Ionian, Dorian, Phrygian, Lydian, Mixolydian and Aolian, using numbers for "strings" found notated in
2800 BC Sumeria.

Unlikely as these circumstances (between 1:81) may seem, evidence can be found in the earliest musical 117 "texts" where a scribal description of a harp-like instrument names its strings using exactly the numbers found 118 119 above in the early number field. This useage of the lowest possible numbers was not restricted to a single modal octave but apparently to a slightly larger range specified as string number 36:40:45:48:54:60:64:72:80. This 120 suggests a knowledge of the derivation given above, of numerical string lengths from first principles, using the 121 harmonic numbers based solely upon the factors of the first three primes, 2, 3 and 5, less than 81. This would be 122 six hundred years earlier than the Nippur tables, where (as already stated) non-harmonic numbers had been 123 124 eliminated from the early number field.

125 Identifying string tones by the numerical string lengths is very natural if numbers were being formed as numerical lengths. Therefore, the earliest depictions for early instruments employing harmonic string numbers 126 127 as initial string lengths, would also have been depicted with tuning knobs to adjust for variations in string properties, due to manufacture, humidity or temperature. The idea of specifying the lengths as 128 129 36:40:45:48:54:60:64:72:80 must have arisen due to a prior *arithmetical* calculation of harmonic intervals using existing small units for an aural discovery of musical intervals. This could have emerged in the prehistory of the 130 131 megalithic period, in which numeracy was exactly based upon the suitable geometrical methods employing a 132 metrology of numbers held within lengths which could be strings.

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Figure 6 Chartres cathedral viewed as employing the wholetone (8:9 transcept) and semitone (15:16 nave & choir),
 potentially signifying thereby Jupiter and Saturn.

137	Part Two: Theological Implications of Astronomical Musicality		
138	From Pa	art One we gather the following key points,	
139	1.	Some significant geocentric time periods stand in harmonic intervals to each other.	
140	2.	The prehistoric numeracy, geared to numbers held as metrological lengths, would have been	
141		well suited to understanding the numerical origins of harmony.	
142	3.	In the 3 rd millennium BC ancient Near East texts appear to refer to the unique minimum string	
143		length sequence predicated on the lowest possible numbers to achieve just tuning scales, these	
144		numbers being less than 81.	
145	4.	Ernest McClain [MCCLAIN 1978] has recovered the Classical form of an ancient harmonic	
146		system used for a harmonic parallelism within narratives, which used the citing of numbers	

148 McClain's matrices are made of large integers which on the outside seem related by just two intervals: from left to right the fifth of 2:3; from bottom to diagonally upwards, the major third of 4:5. In fact the 149 150 mountainous structure of harmonic number "bricks" generates an interval between every pair of bricks 151 throughout the mountain and it is these that will soon locate the lowest limiting number [1440], where the lunar 152 year, Jupiter synod, Saturn synod and other periodicities can all be placed within such a mountain. Thus, each 153 mountain is a lexicon of *distance functions*, each of these forming a harmonic interval between its bricks. But why should astronomical musicality fall into such a scheme at all, since celestial mechanics is not thought of as 154 155 musical? In celestial mechanics, "an **orbital resonance** occurs when two orbiting bodies exert a regular, periodic gravitational influence on each other, usually due to their orbital periods being related by a ratio of two small 156 integers"⁵ but it now seems obvious that the first three harmonic primes will be as dominant in gravitating 157 158 systems as they are in delivering musical harmonies to our ears.

noting harmonic "landmarks".

The tuning view of a limiting mountain is actually misleading since the origin of such a mountain comes directly from the products of three and five found within **harmonic numbers**. Each place on such a mountain has a root product of the powers of three and five such that its height is exactly due to its powers of five and its own location to the right is exactly how many powers of three it contains. For any given limit there can be only one harmonic tone number in each position on the mountain and these are the invariants found within the octave as defined by the octave between the Limit and its halving. To create a mountain for a given limit requires the creation of a table of the cross multiplications of powers of three and five.

⁵ Quoted from Wikipedia on orbital resonance

<sup>Figure 7 To the left: the three steps whereby harmonic matrices are generated from (to the right) a table of the
possible products of three and five, increasing the powers of three to the right and those of five on the diagonal. The
resulting "mountain" of tones looks as if tuned by a set of tone numbers generated using symmetrical fifths
horizontally and symmetrical thirds [4:5] vertically (and their reciprocals), about the given limit, but it is not.</sup>

An example of McClain's more recent work on the Bible concerns the formation of the twelve tribes of Israel, illustrating a progression within numerical limits which has astronomical as well as harmonic relevance. This concentrates upon the root number 45 as 3 times the 15 used (figure 7, left) to develop the limit 60 as 4 times 15. This tripling of a mountains limit causes the brick for D to move one place to the right. The only freedom then remaining is to boost a limit by successive powers of two, to form the sequence 45:90:180:360:720:1440, which appears to offer a harmonic number near to the solar year as 360 days, developed by Plato as his "Calendar" progression, represented below [Timaeus 36c⁶].

⁶ In this passage Plato cites a Pythagorean equivalent involving wholetones of 8:9 and semitones "bent around" into a tone circle an idea which McClain then blends into Plato's Just intonation, from other dialogues.

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179 *Figure 8 The Expansion of an important root limit of* $45 = 3^2 \times 5$ *into a "calendar number" 360 and beyond, purely*

through doubling to generate higher octave limits. The harmonic numbers can be seen as emanating within a tone
circle as invariants based purely on their factors of three and five, as found in the computational mechanism of the
harmonic matrices generating these tone numbers (as figure 7).

183 Whenever a limit contains a single power of five, D will be on the second row of a mountain and this is a very important location for Just tuning, in that the bricks on the rows below and above the limit then provide 184 Just tone alternatives to the Pythagorean cycle of fifths which would also require very large numbers to achieve 185 chromatism. Raising the limit of 9 to the second row, using one power of five to make 45, leads to an improved 186 chromatism, eliminating audible commas⁷ and reducing the size of limiting number required to achieve twelve 187 188 note chromatism. In the case of 45 doubled five times to 1440, twelve tones surround the limit and this appears 189 to have informed the idea of **twelve tribes** developed through Abram as $243 [3^5]$, Isaac as $180 [45 \times 4]$ then Joseph as 125 [5³] who doubled up to 1,000, as Vizier of Egypt, stands above the three Just registers of 1440, 190

191 forming a minor diesis to the cornerstone valued at $1024 [2^{10}]$.

⁷ It takes three major thirds to generate the minor diesis of 128:125, but that would span four rows of the mountain, beyond the range of the symmetrical twins of a D on the second row.

TWELVE Patriarchal Bricks for Harmonic Expansion into Tribes within the Year

Joseph stands above the Year Calendar where he becomes ONE Thousand, revealing all the tonal ratios of **the whole mountain** in a decimal "vectorization"

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Figure 9 Joseph is cornerstone to the idea of twelve tribes who will become saviour of the descendants of Isaac =
180 and hence form the twelve tribes, as ab = 45/32. This cornerstone of 128, having no powers of three and five, is
raised by the minor diesis [128:125] to 125, then doubled thrice to 1000. The twelve tribes represent a superior Just
chromatism whilst the saviour, now 1024, stands opposite the Plato's tyrant number [729] on the base of the

mountain for 1440.

The problem with such an explanation appears to be why the authoring individuals should seek to drive the narrative of Genesis using a hidden framework of harmonic facts, here found within limits based upon 45. Is this simply a desire that the text preserve harmonic knowledge in occult form, or is there something more important underlying these mountains, about twelve tribes, or tones, such as the characterisation of a divine world of astronomical musicality? Ernest McClain gives a character explanation of 1440 as being 'little Adam'

203 read as Hebrew A.D.M meaning (the number values 1.4.40 as) 1440, justifying 720 as middle of the paired Great 204 and Small double octave, perhaps using 45 to displace Semitic gods, particularly the Assyrian and Babylonian 205 goddess ISHTAR [= 15] as the tonal root of D. We know for instance that the Jews followed a lunar calendar and that there are twelve lunar months within a solar year. The Pentateuch opens with six days of creation (the 206 1:2:3:4:5:6 of the *Senarius*⁸) and rests on the seventh day, a non-harmonic prime. The planetary designations of 207 208 seven days in the week are lacking in semitic usage (unlike the Indo-European equivalent observance of the 209 week) perhaps because it was idolatrous, implying a denial of "other gods" as the planets, who are merely agents of harmony like the demiurge. However, the seven day week has only one astronomical sponsor which is the 210 211 god of time, Saturn, whose synod is divided perfectly by it (as 378 divided by 7 days is 54 weeks). The synod 212 of Jupiter is similarly commensurate with seven days as 57 weeks, and the two giant planets are then shown to 213 be in ratio as 18:19 when seen through their difference of 21 days or three weeks. Such things were known when 214 people spent time under the skies and counted the time between events.

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It is time to find a reason for there being two intervals [8:9 and 15:16] between the lunar year and the synods of Jupiter and Saturn, respectively. The key lies in dividing the lunar month by 80 parts (each of 0.369 days) after observing that, the next-door-but-one neighbours on McClain's mountain rows form a whole tone [8:9] (see above figure) and that, in the limit for 1440 above, the left hand brick to 1440 is 960, which is 80 times 12 (a lunar year equalling 12 lunar months). 81 of these units (of 0.369 days) equals 30 revolutions of the earth (relative to ecliptic). That is, the duration of a lunar month (29.53059 days) relative to that of a month of sidereal days (each 0.997283 days long) form the interval called the synodic comma [80:81] generated by Just tuning.

Figure 10 Location of the synods of Jupiter and Saturn as intervals to the Lunar Year of twelve months, within the
 matrix of 1440 equal to a limit of eighteen lunar months.

⁸ This a reuse by Ernst Levy [LEVY, 1992], one of McClain's mentors, of the lambic Metre - a verse of six feet, especially an lambic trimeter.

Since next-door-but-one neighbours are 8:9, the Jupiter synod must then be 1080 of these synodic units whilst Saturn becomes the "cornerstone" of the mountain for 1440 as 1024 *syntonic* units (see next). Hence perhaps, the fuss about Joseph and eventually Jesus as "the stone the builders rejected". McClain identifies the cornerstone of Isaac's [180] patriarchal matrix as being Reuben, himself a saviour through saving Joseph from his other brothers, each brother the notional head of twelve future tribes, each created through Joseph's inspired transition to Egypt, equal to one thousand syntonic units.

The narrative construction is preserving an astronomical matrix relevant to a cornerstone of Saturn [1024], whose designation of the number seven is a prime number that *will not mix* with the other harmonic numbers, especially in a harmonic context, whilst also being the ancient designation of an extra divine part, greater than ordinary mortals and hence heroic in an age of god-kings and royal cubits of 12/7 feet.

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237 part of its synodic month as a new unit which enables an astronomic musicality to appear within the limit of 1440,

overseen by 1000 but also, though invisibly, the eclipse year [1875/2] and another minor diesis [128:125], this time
 to the lunar year.

240 In the astronomical version of 1440, Joseph as 1000 units would be the synod of the planet Uranus of 369.66 days, an outer planet only visible to the naked eye when its position is known. Its role within this matrix 241 is notable for its ability, as 1000, to make all the bricks of the matrix "read" like decimals in place notation 242 243 without any ancient ability to use aggregates in a fractional notation. Joseph stands at the same harmonic distance 244 [25:27] to the Jupiter synod, as the lunar orbit does to the lunar month. Astronomically it is Uranus that is related 245 to the Saturn synod as the minor diesis [128:125], as is the eclipse year (shown black and an illegal fraction 246 [937.5 units] in this context, as half of a yet to appear 1875) now related to the lunar year. Saturn, Joseph and 247 the eclipse year stand outside the twelve tonal tribes surrounding ADM as 14.4.0, whilst the lunar year and Jupiter synod are each one of the twelve tribal bricks, merely actors manifesting God's harmony. Only 248

²³⁶ *Figure 11 Just as the early number field offers a syntonic comma of 80:81, so also does the moon provide an 80th*

cornerstone Saturn can provide the 12th note within the tone circle [right of figure 10], though 1024 is not symmetrical relative to both high and low D but, being an approximation to octave's geometrical mean [the square root of two] opposes D in the logarithmic world of tone perception to give the twelfth tone of a-flat, if and when required.

It seems significant that the two main aspects of the moon, the eclipse year and the lunar year, are shown 253 254 here related to three rows of a practical **pentatonic musicality**, having twelve symmetrical tones and chromatic options via Just and Pythagorean alternatives for the same tones and with the Saturnine 13th tone. McClain's 255 story may be substantiated by resorting to the definite astronomical correspondences to harmonic intervals, based 256 257 as it is on a relatively small number set [1440], similar in scale to four years of day counting [1461 days]. However, the idea that celestial mechanics could ever have cooperated with such astronomical arrangements 258 259 might seem implausable to a modern scientist who prefers only natural solutions to mechanical problems, hence based upon physical forces with no design or designer. (Giving significance back to geocentric and synodic time 260 261 periods perhaps grates with the momentous move to heliocentric dynamics, responsible for so much scientific 262 progress.)

263 However there is in principle no difficulty here for, in celestial mechanics, the solution of the n-body 264 problem (which gravitating systems like our earth, sun, moon and planets present, from multiple centres,) can 265 often reduce to various special cases that are solvable (that is integrable) to yield interval ratios between the astronomical periodicities, these based upon "small whole number ratios"⁹, providing certain conditions prevail 266 267 between the bodies such as repeated points of symmetry in the configuration of celestial bodies. The ancient 268 tuning matrices proposed by McClain do express a perfect symmetry between their tone sets, using small harmonic numbers. Symmetry in tones could represent the mutual gravitational influences between celestial 269 270 bodies. It may also be true that modern celestial mechanics fails to see the considerable power given to harmonic 271 numbers within the early number field, and so fails to report it. One reason for filtering out celestial harmonics 272 might be the long-standing belief in a "harmony of the spheres", prevalent in the pre-scientific thinking from 273 Plato to Kepler: has modern astronomy become "harmony averse"?

274 SUMMARY

- We have seen that harmonic intervals exist between celestial time periods and that these intervals could have inspired a characterisation of the planetary world as being ruled by harmony, hence the enduring notion of
- a harmony of the spheres.that is of the planetary periods

⁹ "In celestial mechanics, an **orbital resonance** occurs when two orbiting bodies exert a regular, periodic gravitational influence on each other, usually due to their orbital periods being related by a ratio of two small integers." Wikipedia

The ideal circumstance for this step to have been taken is the megalithic period of the 5th and 4th millennia BC, when monuments were built in which (a) time periods were counted as lengths and (b) lengths were compared using right triangles. Such triangles could reveal the invariant interval between time periods in their slope angle and via dividing by their differential length to obtain the superparticular (n:n+1) ratio for the interval, the *megalithic yard* being one such differential length, obtained over a three year count.

Once we allow for the development of such a pre-arithmetic form of what is nonetheless calculation, as being performed in prehistory; one can see that astronomy, a study of number and or musical intervals, could have preceded the 3rd millennium's own contributions to these subjects in the ancient Near East.

Perhaps the greatest contribution of the ancient world has been literary in the form of mythic narratives. Ernest McClain has used his understanding of Plato's system of harmonic analysis to see parallels being made through mention of definite numbers within such texts. Numbers can generate entire harmonic matrices (or holy mountains) and these then used by ancient authors to ensure that their stories reflected a harmonic world evidently considered sacred or "higher".

The phenomenon we call Just tuning can be seen as implicit within the early number field and explicit in the synods of Jupiter and Saturn relative to the lunar year whose twelfth part, the synodic month, provides the syntonic comma [80:81] relative to thirty earth rotations, within the matrix limit 1440, Plato's calendar octave limit.

This journey of possible origins, from harmonic parallelism within myth to an actual celestial harmonics, is explainable through celestial dynamics which, under certain special cases (of balanced forces) reduces chaotic n-body situations to develop regular periodicities between the gravitating bodies. Whether the resulting harmonics are divinely ordained is not relevant to the question of whether the ancient world saw such celestial harmonies as expressive of a divine principle when forming their related cosmogonies and theologies.

301 Appendix 1: Astronomical Periods and their Matrix Equivalents

Planetary Period	Practical Definition	Period in days	in matrix units ideal:actual:error
Lunar Month	Cycle of phases, full- to-full	29.53059	80; 80; n/a
30 Sidereal Days	A sidereal month	29.9185	81; 81.05; 1/76
Sidereal Day	Rotation of the earth relative to the ecliptic	0.99728 <u>3</u>	2.7; 2.702; 1/1584
18 lunar months	Limit for the 1440 matrix	531.55062	1440; 1440; n/a
Lunar Year	Twelve lunar months between thirteen full moons	354.367	960; 960; n/a
Jupiter Synod	Time between retrograde loops	398.88	1080; 1080.58; 1/1837
Saturn Synod	Time between retrograde loops	378.09	1024; 1024.27; 1/3840
Uranus Synod	Invisible to naked eye astronomy	369.66	1000; 1001.43; 1/700
Eclipse Year	Time between eclipses of the moon at same node	346.62	937.5; 939.01; 1/620

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304	BIBLIOGRAPHY			
305	BLAKE, Anthony, The Intelligent Enneagram, Boston, Weiser: 1996			
306 307	DOUGLAS, Mary, Thinking in Circles: An Essay on Ring Composition (The Terry Lectures), New Haven, Yale University Press: 2010			
308	HEATH, Richard			
309	Matrix of Creation: Sacred Geometry in the Realm of the Planets, Vermont: Inner Traditions, 2004			
310 311	Sacred Number and the Origins of Civilization: The Unfolding of History through the Mystery of Number, Vermont: Inner Traditions, 2007			
312 313	Precessional Time and the Evolution of Consciousness: How Stories Create the World, Vermont: Inner Traditions, 2011			
314 315	Sacred Number and the Lords of Time: The Stone Age Invention of Science and Religion, Vermont: Inner Traditions, 2014			
316 317	HEATH, Richard and Robin, The Origins of Megalithic Astronomy as found at Le Manio (Online at academia.edu)			
318	LEVY, Ernst, A Theory of Harmony, Albany, SUNY: 1985			
319	MCCLAIN, Ernest,			
320 321	Myth of Invariance: The Origins of the Gods, Mathematics and Music from the Rg Veda to Plato, New York, Nicolas-Hays: 1976			
322	The Pythagorean Plato: Prelude to the song itself, New York, Nicolas-Hays: 1978			
323				