

The Algorithmic Origins of Diatonic Tuning Systems

The diatonic or natural scale, consisting of five whole tones and two *opposed* semitones, is most familiar today in the white notes of the piano [Apel. see *Diatonic*]. On the piano this would be called C-major, which imposes the sequence of tones (T) and semitones (S) as T-T-S-T-T-S in which the initial and final tetrachords are identically T-T-S, leaving a tone between F and G, the two fixed tones of the Greek tetrachordal system

The diatonic scale is ... an abstractum; for all we have is five tones and two semitones a fifth apart [until] we fix the place of the semitones within the scale, thereby determining a definite succession ..., [and] we create a mode. [Levarie. 213].

One can see that the tones are split into one group of two (T-T) and one group of three (T-T-T), by the major diatonic, so that the semitones are opposed (B-F) towards the tonic C as in figure 1.

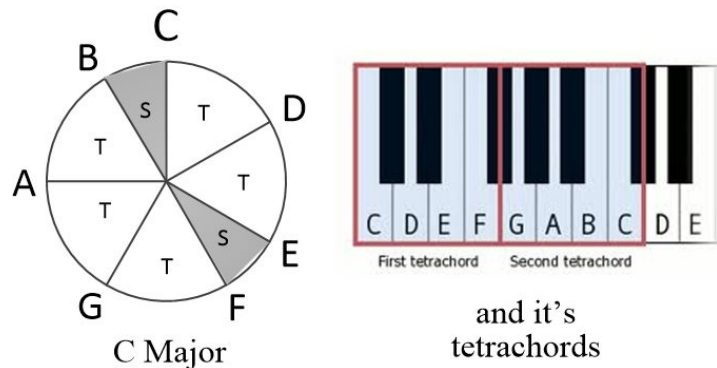


Figure 1 Tone circle and tetrachords for C-Major also called the modal scale of Ionian

Letters such as C are called note classes so as to label the tones of a diatonic scale which, shown on the tone circle, can be rotated into any key signature of twelve keys including flattened or sharpened notes, shown in black in figure 1. We will first show how these black notes came about algorithmically, due to two aspects of common usage.

The note classes arose from the need of choral music to notate music so that it could be stored and distributed. When we “read music” today, the tablature consists of notes placed within a set of five lines with four gaps, and two extendable areas above and below in which only seven note classes can be placed, seven being the number of note classes in the modal diatonic and the number of white keys on the keyboard, which is the other aspect of usage.

Present day keyboards and the note letters

One can see that the note classes (A-B-C-D-E-F-G) do not correspond with the white keys of the keyboard correspond to C-Major (C-D-E-F-G-A-B) and this can only be because the first keyboards, used to control groups of bells, or organ pipes, and then clavichords and harpsichords, were naturally playing the major diatonic, which today starts with our first note C. The notational system of classes was developed to help choirs learn plainsong as do-re-me-fa-sol-la after Guido of Arezzo, who lived from 992 until after 1033¹), regarded as the father of modern musical notation but not of the note names. Guido's system forms the basis of the solfeggio system, our "do-re-me-fa-sol-la-si-do", still widely used in non-English speaking countries. The note Do is invariably the start and end of our C major scale and hence, solfeggio was remarkable compatible with the later notion of key signatures in which each key is rooted in the major scale.

When the church wanted to play different modal scales in keyboard instruments, black notes were added between the white notes of existing keyboards². These black notes arrived in a distinctive order as one tries to play Greek modal scales from a common starting point such as C. They came to be designated sharp or flat, which implies a process by which they came into existence. In the case of the Dorian, the note B-flat will replace B and so a black key was added between A and B. Whilst the black notes appeared naturally within keyboards, when church modes needed to be performed, modern music has abandoned those scales exactly because the keyboard and musical notation treated the major scale as the primary scale. The eponymous system of *key* signatures, starting from any key in the major scale including the black notes, does not find note C as first key of the keyboard's notes. Keyboards evidently started with C, having only the white notes we call C major. So why should A be our first note class rather than C?

The idea of notes as letters was first proposed by Boethius (c. 480-524) who started with the A of the alphabet but having sixteen note letters – that is, not constraining himself to a single octave scale. "A" was nothing more than a label for

¹ He published *Micrologus*, 1026. The original letters used by Guido were the "ut-re-mi-fa-so-la" syllables are taken from the initial syllables of each of the first six half-lines of the first stanza of the hymn *Ut Queant Laxis* in which the first syllable of each line was sung in the order of the major diatonic. Ut was changed to Do by Giovanni Battista Doni: He convinced his contemporaries to make the change by arguing that "Do" was easier to pronounce and was an abbreviation for "Dominus," the Latin word for The Lord, who is the tonic and root of the world.

² On early instruments now only existing in art. Church influences led to keyboards being played within art by angels, choristers or saints.

the lowest note but Notker Babulus (d. 912) first used A to refer to what we now call C, the home key of the white note major scale. But Boethius was eventually followed since, in his system of note letters, the initial tetrachord after A was tone-semitone-tone (T-S-T), which is the tetrachord of the symmetrical Dorian scale T-S-T-T-T-S-T. The Dorian was probably chosen by Boethius because this is the scale of the Pythagorean heptachord, resulting from tuning by pure fifths and fourths within the octave. A is the first note below C that can play that tetrachord on the white notes of the keyboard. Choosing A locates D as the white note centre of symmetry amongst the population of black notes on the keyboard and D is the natural Dorian key having T-S-T *either side of it*, with A below and G above.

This gives us a reasonable historical likelihood, with three important initial conditions,

1. Boethius invented note letters starting with the symmetrical tetrachord of Dorian, whilst
2. it was the major scale that ruled early western music, upon which the solfeggio registered its notes as do-re-me-fa-sol-la-si-do, and
3. early keyboards had only the major scale as white notes but developed black notes to allow modal scales to be played.

Having created chromatic keyboards, there was a natural pattern of white and black notes which did not follow the idea of the first note do being C. The Pythagorean heptachord, created using fifths and fourths when tuning lyres, harps, clavichords and harpsichords, evidently started Boethius' note letters, and this meant C must move from being the first white note of the keyboard to being the third (after the semitone of a tetrachord T-S-T of the notes A-B-C-D). Because Dorian has both an initial and final tetrachord of T-S-T, this created the present keyboard which is symmetrical about D.

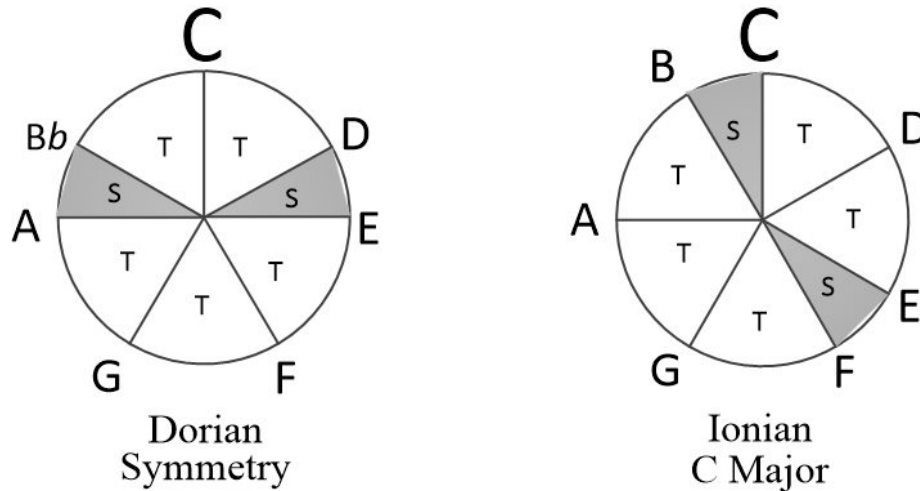


Figure 2 Defining modal scales in C, other than the major diatonic, requires the black notes of the keyboard and new chromatic note classes in tablature

The System of Key Signatures

The key signature system was then conceived as being from white key C, since the note classes of C-Major key are all unmodified letters, with no sharps or flats. However, whilst the major diatonic can be played starting from different keys this can be hard to learn since more or less black notes replace the white notes of the major diatonic in different keys. For example, if one is to play the major diatonic sequence of T-T-S-T-T-T-S in the Dorian-in-C tone circle of figure 2 (left), then one would start with black note B \flat and hence the key B \flat major. Similarly, if you wanted to play the Dorian on the right-hand tone circle of figure 2, that is in C-major, you would start with D and could still play all white notes.

This gives us a clue as to how, whilst key signatures filled the key-board with major scales, the modal scales were still available within each key once black notes were added to the keyboard. The primacy of the major scale probably drove the adoption of key signatures and the evolution of modern practices of modification within key signatures called minor, seventh, augmented and so forth. These modifications have the advantage of being similarly available in each key signature, modifying the major diatonic in a lesser fashion than playing other modal scales draws on many of the different tonal features found in the antique world of scales. Once the fingering for a given key was mastered, similar melodic and chordal adjustments could be similarly found in every key, allowing *transpositions* of musical ideas between the key signatures.

Another feature of key signatures is made clear in tablature where key signatures are associated with a number of sharpened or flattened note classes that then run on along the staff's (a.k.a. the staff's) lines and gaps. This is supplemented by accidentals before notes, to achieve note classes not within the major key. Perhaps more importantly, as with the Dorian in C of figure 2, the key signatures starting from C develop increments of one sharp or one flat to form a natural sequence as in figure 3.

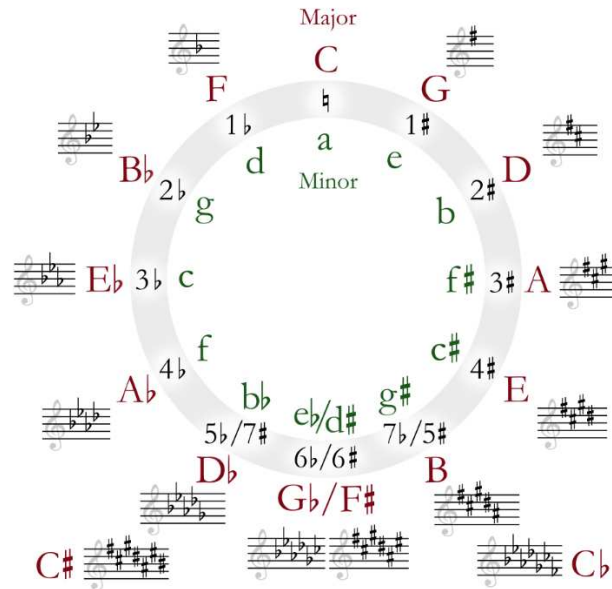


Figure 3 The cumulative character of sharps and flats associated with key signatures and the note class of their minor variation. Wikipedia user: just plain Bill

To travel from C major to the key with just one flat, the key must start a descending fifth from C, that is in F-major. Similarly, the key with a single sharp is G-major, an ascending fifth from C. This mechanism of travel, around the circle of note-key classes continues algorithmically when adding two flats with Bb-major being a descending fifth from F; and adding two sharps in D-major, moving the tonic through an ascending fifth from G. This continues until the twelve key-signatures are exhausted³. This process of generation, moving the tonic through fifths requiring new flats or sharps, impacts how the black note classes are called since every player relates to the black notes primarily through the key that requires them as chromatic⁴ flats or sharps.

³ apart from special keys that are different ways of looking at one of these twelve keys.

⁴ The chromatic notes were initially the five notes not required by the major scale, but chromatic music more loosely means music using twelve notes to the octave rather than just seven.

This naming of chromatic notes is particularly clear when non-musicians try to name them “wrongly”, that is not according to their emergence due to the key signature system. The system of keys is an algorithm for ordering the keys by their complexity of sharps and flats, and the consequent separation of the keys so arranged (by descending and ascending fifths) is fundamental to the process whereby the old Greek-style scales have been side-lined by modern music through the convenience of playing in the major scale starting with each possible key of the keyboard.

Of course, the evolution of such a system would never have been possible in practice without the attempt to temper the tones and semitones away from being pure numerical ratios so that each key should have uniform tones and semitones of one defined size. This search for equality of tones at the expense of exactitude to the ratios of ancient tuning systems was crucial to being able to write most of the recent Classical music within the key signature system. Our key signature system today works perfectly, based upon equal temperament in which each twelve chromatic semitones are equal in ratio being to the twelfth root of the 2 of octave doubling.

One is led to infer four influences upon the eventual system of key signatures:

Firstly, they are a potential held between the twelve chromatic notes arising in modal scales

Secondly, the note letters innovated by Boethius and alternate solfeggio notation of Guido directed musical practices dominated by the major scale

Thirdly, the early keyboard instruments, with white keys but then black chromatic keys placed A before C in Boethius’ final tetrachord for D, the natural axis of symmetry for the keyboard, corresponding with the Heptatonic.

Finally, the tempering of strings using beats established a non-theoretical basis for equal temperament.

